Discounting and Catastrophic Risk Management

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ABSTRACT

The goal of this paper is to analyse implications of discounting on catastrophic risk management. For example, how can we justify mitigation efforts for expected 300-year flood that may occur next year. The discounting is supposed to impose time preferences for investments to resolve this issue, but this view may be dramatically misleading. We show that any discounted sum of values can be equivalently replaced by undiscounted sum of the same values with random finite time horizon. The expected duration of this "stopping" time horizon for standard discount rates obtained from capital markets does not exceed a few decades and therefore such rates may significantly underestimate the net benefits of long-term decisions. The alternative undiscounted random stopping time criterion allows inducing discounting focusing on arrival times of catastrophic events rather then horizons of market interests. In general, induced discount rates are conditional on the degree of social commitment to mitigate risk. Random extreme events affect these rates, which alter the optimal mitigation efforts that, in turn, change events. This endogeneity of the induced discounting restricts exact evaluations necessary for using traditional deterministic methods. The paper provides insights in the nature of discounting that are critically important for developing robust catastrophic risk management strategies by using stochastic optimization methods.

KEY WORDS: Extreme events, stopping time, catastrophic risks, discounting, investments, stochastic optimisation, risk measures.

INTRODUCTION

Implication of extreme catastrophic events for justifying long-term investments is a controversial issue. How can we justify investments, which may possibly turn into benefits only over long and uncertain time horizons in the future? This is a key issue for catastrophic risk management. For example, how can we justify investments in climate change mitigations, say, in flood defense systems to cope with foreseen extreme 1000-, 500-, 250-, and 100- year floods?

The discounting is supposed to impose necessary time preferences for investments, but this view may be dramatically misleading. There are several possibilities for choosing discount rates (see, for example, the discussion in (Arrow et al., 1996; Manne, 1999; OXERA, 2002; Toth, 2000). Most traditional models assume the discount factor is the same as the rate of return in capital market that a person could receive by investing money elsewhere over the given period of time. This discounting relates to the geometric or exponential discount factor $d_t = (1+r)^{-t} = e^{-\ln(1+r)t} \approx e^{-rt}$ (for small r) connected with a constant rate r of returns from the markets. A large number of studies demonstrate that the constant geometric discounting is often inappropriate. Thus, many authors view the hyperbolic discounting as a more natural way to define the intertemporal preference structure of the decisions. In hyperbolic discounting, valuations fall rapidly for a few first time periods and then fall much slower. The value of future rewards under hyperbolic discounting is much lower than under exponential discounting. Gasgupta and Maskin (2004) propose an example offered by O'Donoghue and Rabin (1999) according with hyperbolic discounting: The humans when offered a painful treatment for 7 hours in February or for 8 hours in April, most frequently choose the earlier date. But as this date approaches, they change their minds and postpone the pain until later although it will be greater. Strotz (1956) discusses similar example with people deciding to lay aside money for future. However as time goes, they change their minds as if they become less patient.

In general, the choice of discounting rate equal to market return rate is linked with the assets having a lifespan of only a few decades. This may substantially reduce the impacts that investments may have beyond these intervals. Namely, in what follows we show that any discounted sum $\sum_{t=0}^{\infty} d_t V_t$ of expected values $V_t = Ev_t$ for random variables (r.v.) v_t, $d_t = (1+r_t)^{-t}$, $t = 0,1,...$, under constant and declining discount rates r_t equals the average undiscounted random sum $E\sum_{t=0}^{2} \neg_t$ with a random stopping time ^{2} defined by discount factors d_t as probability $P[^2 \geq t] = d_t$. Therefore, discount factors d_t can be associated with the occurrences of "stopping time" random events determining a finite internal discount-related horizon $[0,^2]$. The expected duration of ^{2} and its standard deviation $\pm$ under modest market interest rates of 3.5% is approximately 30 years, which may have no correspondence with expected, say, 300-year extreme events. Conversely, any stopping time associated with the first occurrence of a random event induces a discounting. A set of random events, e.g., 1000-, 500-, 250-, and 100- year floods, induces discounting with time-declining discount rates.

Another serious problem (Heal and Kristrom, 2002; Newel and Pizer, 2000; Weitzman, 1999) arises from the use of the traditional expected value Er and the discount factor $(1+Er)^{-t}$ that implies additional significant reduction of future values in contrast to the expected discount factor $E(1+r)^{-t}$, since $E(1+r)^{-t} >> (1+Er)^{-t}$. An appropriate exact interest rate is especially difficult to define when decisions involve time horizons beyond the interests of the current generation. If future generations are not present in the market, e.g., long-term environmental damages are not included in production costs and the market interest rates do not reflect the preferences of future generations.

Debates on proper discount rates for long-term intergenerational problems have a long-standing history (Arrow et al., 1996; Toth, 2000). Ramsey (Ramsey, 1928) argued that applying a positive discount rate r to discount values across generations is unethical. Koopmans (Koopmans, 1966), contrary to Ramsey, argued that zero discount rate r would imply an unacceptably low level of current consumption. The constant discount rate has only limited justification (Chichilinskii, 1996; Frederick et al., 2002; OXERA, 2002; Toth, 2000). Cline (Cline, 1999) argues for a declining discount rate: 5% for the first 30 years, and 1.5% later. There have been proposals for other schedules and attempts to justify the shape of proper decline. Papers (Newel and Pizer, 2000; Weitzman, 1999) show that uncertainty about r produces a certainty-equivalent discount rate, which will generally be declining with time. Weitzman (Weitzman, 1999) proposed to model discount rates by a number of exogenous time dependent scenarios. He argued for rates of 3 – 4% for the first 25 years, 2% for the next 50 years, 1% for the period 75–300 years and 0 beyond 300 years. Newell and Pizer (Newel and Pizer, 2000) analyzed the uncertainty of historical interest rates by using data on the US market rate for long-term government bonds. They proposed a different declining discount rate justified by a random walk model. Chichilinsky (Chichilinskii, 1996) proposed a new concept for long-term discounting with a declining discount rate by attaching some weight on the present and the future consumption. All these papers aim to derive an appropriate exogenous social discount rate that is in general practically impossible for catastrophic risks.

Catastrophic events pose new risk management challenges. They often create so-called endogenous, unknown (with the lack and even absence of adequate observations) and interdependent systemic risks (Arrow, 1996; Arrow et al., 1996; Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermolieva et al., 2003; Heal and Kristrom, 2002). As a consequence, catastrophic risks require development of spatially explicit catastrophe models (Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000; Weitzman, 1999). In combination with this models, the concept of random stopping time criteria allows to induce social discounting that focuses on arrivals of catastrophic events rather then the lifetime of market products. Since risk management decisions affect the occurrence of disasters in time and space, the induced discounting may depend on spatio-temporal distributions of extreme events and feasible sets of decisions. This endogeneity of induced spatio-temporal discounting calls for the use of stochastic optimization methods, which allow also to address the variability of discounted criteria by using random value $\sum_{t=0}^{2} v_t$ even for deterministic v_t, $t = 0,1,...$. Misperception of induced by stopping time discounting provokes catastrophes.

STANDARD DISCOUNTING vs STOPPING TIME

The standard exponential discounting, or geometric discounting for discrete-time models, is a dominant approach used by leading economists (Luenberger, 1998) and other analysts (Haurie, 2003; Heal and Kristrom, 2002; Manne, 1999; Newel and Pizer 2000; Nordhaus and Boyer, 2001; Toth 2000) to justify long-term programs. Expected outcomes of a program are defined as s stream of values V_0, V_1, …, $V_t = Ev_t$. Then alternative programs are compared by total value

$$V = \sum_{t=0}^{\infty} d_t V_t \,, \tag{1}$$

of V_0, V_1, … discounted to present time $t = 0$, where d_t are discount factors, $0 < d_t \le 1$. For the dominant geometric discounting, $d_t = d^t = \exp(t \ln d)$, $0 < d < 1$. If V_t is the instantaneous welfare function, then V evaluates the welfare of an infinitely living society discounted to the initial time $t = 0$. The following Proposition 1 and Remark 1 show that the use of discounted criterion (Eq.1) can be rather misleading.

The stream of values V_t, $t = 0,1,...$, can represent an expected cash flow stream of a long-term investment activity. In economic growth models and integrated assessment models (Manne, 1999; Nordhaus and Boyer, 2001; Toth, 2000) the value V_t represents welfare $V_t = \sum_{i=1}^{n} \sim_i u_i(c_i^t)$ of a society with representative agents $i = \overline{1,n}$, utilities u_i, consumptions c_i^t and welfare weights $\sim_i$.

Natural selection theory treats Eq.1 as Darwinian fitness (Sozou and Seymour, 2003), where discount factors d_t are associated with hazard rates of an environment (Example 2).

The infinite time horizon in Eq.1 creates an illusion of truly long-term analysis. Propositions 1 and 2 show that in fact evaluation by Eq.1 accounts only for values V_t from a finite random horizon $[0,\tau]$ defined by a random stopping time τ with the discount-related probability $P[\tau \geq t] = d_t$. Let us consider so far the simplest case of geometric discount factors.

Proposition 1. Consider standard geometric discount factors with constant discount rates r, $d_t = d^t$, $t = 0,1,...$, $0 < d < 1$, $d = 1/(1+r)$. Let $q = d$, $p = 1 - q$, and τ be a random variable with the geometric probability distribution $P[\tau = t] = pq^t$. Then $d_t = P[\tau \geq t]$ and

$$\sum_{t=0}^{\infty} d_t V_t = E \sum_{t=0}^{\tau} V_t .$$

Proof of this proposition follows from the proof of general Proposition 2.

Remark 1 (Random stopping time horizon). The same fact (Proposition 2) is true for general discount factor d_t. We can think of τ as a random "stopping time" moment associated with the first occurrence of a "killing" catastrophic or "stopping time" event. The probability that this event occurs at $t = 0,1,...$ is p and pq^t is the probability that this event occurs first time at t. The expected duration of τ, $E\tau = 1/p = 1 + 1/r \approx 1/r$ for small r. A standard approach to the choice of the discount rate r is to use market interest rates (Luenberger, 1998; Toth, 2000). Therefore, for r related to the market interest rate of 3.3%, $r = 0.033$, the expected duration of the stopping time horizon is $E\tau \approx 30$ years, i.e., this rate orients the policy analysis on an expected 30-year catastrophic event. Certainly, this rate has no relation to how society has to deal with a 300-year flood, i.e., a flood with an expected arrival (stopping) time of 300 years. Proposition 1 shows that the discount rate r can be interpreted as a killing rate (Haurie, 2003) which makes the life expectancy of an infinitely living agent equal to $1 + 1/r$ years. The implications of Proposition 1 for long-term policy analysis are rather straightforward.

Thus indeed any discounted deterministic sum in Eq.1 equals the average undiscounted random sum $\sum_{t=0}^{\tau} V_t$ of the same values V_t. Therefore, the discount factor $d_t = d^t$ induces an "internal" discount-related time horizon $[0,\tau]$ with the geometrically distributed τ. Conversely, any geometrically distributed τ and the criterion $E\sum_{t=0}^{\tau} V_t$ induces the geometric discounting in the sum $\sum_{t=0}^{\infty} d_t V_t$. Proposition 2 demonstrates that this is true for general discount factors.

Remark 2 (Variability of NPV, connections with dynamic CVaR). Disadvantages of the standard deterministic discounted criterion Eq.1 are well known (Luenberger, 1998). In particular, it does not reveal the temporal variability of cash flow streams. Two alternative streams may easily have the same NPV despite the fact that in one of them all the cash is clustered within a few periods, but in another it is spread out evenly over time. This type of temporal heterogeneity is critically important for dealing with catastrophic losses which occur suddenly as a "spike" in time and space (Ermolieva and Ermoliev, 2005).

The criterion $E\sum_{t=0}^{\tau} V_t$ allows to address distributional aspects and robust strategies (Ermoliev and von Winterfeldt, 2010) by analyzing the random variable $\sum_{t=0}^{\tau} v_t$, $V_t = Ev_t$ (even for deterministic $v_t = V_t$), e.g., quantiles defined as maximal $y = y_\alpha$ satisfying safety constraints

$$P[\sum_{t=0}^{\tau} v_t \geq y] \geq \alpha .$$

Equivalently, it is well known that y_α maximizes the concave function (see discussion in Ermoliev and von Winterfeldt, 2010)

$$y + \alpha^{-1} E \max\{0, \sum_{t=0}^{\tau} v_t - y\} .$$

Therefore, if variables v_t depend on some decisions x, then the maximization of function

$F(x)+[y+¥^{-1}E\max\{0,\sum_{t=0}^{²}v_t-y\}]$

allows easy control of quantiles of $\sum_{t=0}^{²}v_t$. The optimal value of this function defines dynamic versions (Ermolieva et al., 2003; OXERA, 2002) of the CVaR (Conditional Value-at-Risk) risk measure (Rockafellar and Uryasev, 2000).

Example 1 (Catastrophic Risk Management). The implications of Proposition 1 for long-term policy analysis are rather straightforward. Let us consider an important case. It is realistic to assume (OXERA, 2002) that the cash flow stream, typical for investment in a new nuclear plant, has the following average time horizons. Without a disaster the first six years of the stream reflect the costs of construction and commissioning followed by 40-years of operating life when the plant is producing positive cash flows and, finally, a 70-year period of expenditure on decommissioning. The flat discount rate of 5%, as Remark 1 shows, orients the analysis on a 20-year time horizon. It is clear that a lower discount rate places more weight on distant costs and benefits. For example, the explicit treatment of a potential 200-year disaster would require at least the discount rate of 0.5% instead of 5%. A related example is investments in climate change mitigations to cope with potential climate change related extreme events. Definitely, a rate of 3.5%, as often used in integrated assessment models (Toth, 2000), can easily illustrate that climate change does not matter.

Example 2 (Darwinian fitness). It is likely that humans and animals intuitively discount future relying on stopping time argument. Ramsey (Ramsey, 1928) had introduced discounting, first of all, as a mathematical device ensuring the convergence of infinite horizon cumulative values. Its various explanations supported by empirical studies were proposed afterwards some of them suggesting that humans and animals place less weights on the future then on the present (see discussion in Sozou and Seymour, 2003). A reason is that future rewards run more risk of disappearing. Hence, they should be discounted, where the discount rate is the hazard rate. For example, evidence from selection experiments indicates the existence of a trade-off between short-term and long-term fertility, i.e., the existence of life-history strategy that discounts the future. In other words, natural selection puts a premium on immediate reproductivity. Accordingly, an animal can be treated as a rational optimizer maximizing its Darwinian fitness, that can be taken to be equivalent to maximizing the expected number of offsprings, i.e. preventing “killing” events. In a simple case, fitness is defined (Sozou and Seymour, 2003) then as integral $F=\int_0^\infty m(t)s(t)dt$, where $m(t)$ is the expected rate of reproductive output at age t if the animal survives to that age, and $s(t)dt$ is the probability of surviving to age t . It is highly unlikely that an animal is able to learn discount factors (probability density $s(t)$) in order to maximize the Darwinian fitness. The equivalent distribution free stopping time criterion requires observations of only lifetime intervals ² , which can be easily used for adaptive adjustments of life-history strategies.

TIME DECLINING DISCOUNT RATES: SETS OF CATASTROPHIC EVENTS

Proposition 1 can be generalized to time-varying discount rates and general stopping time moments. The stopping time ² is defined as a random variable $² \in \{0,1,...\}$, such that event $\{² \le t\}$, $t=0,1,...$ does not depend on values $¬_{t+1},¬_{t+2},...$. For example, ² is the arrival time of the first event from a set of extreme events such as 1000-, 500-, 250- and 100- year floods. The following Proposition 2 shows that any discount factors d_t induce a stopping time and, the other way round, any stopping time ² induces discount factors with (in general) time-varying discount rates (see also Ermolieva et al., 2003). Variables d_t and ² are connected by the relation: $E\sum_{t=0}^{²}¬_t=\sum_{t=0}^{\infty}d_tV_t$, $V_t=E¬_t$. A set of events induces in general discount factors d_t having the time-varying discount rate (Example 3) which is determined for $t\to\infty$ by the least probable event.

Proposition 2. Consider a discounted sum $\sum_{t=0}^{\infty}d_tV_t$, $d_t=(1+r_t)^{-t}$, where r_t is an increasing positive sequence, $V_t=E¬_t$. Then there is a stopping time ² such that $P[² \ge t]=d_t$ and

$$\sum_{t=0}^{\infty}d_tV_t=E\sum_{t=0}^{²}¬_t \tag{2}$$

Conversely, let $E|v_t|$ is uniformly bounded. Then, for any stopping time ²

$$E\sum_{t=0}^{²}¬_t=\sum_{t=0}^{\infty}d_tV_t,\ d_t=P[² \ge t], \tag{3}$$

where V_t is conditional expectation: $V_t=E[¬_t\,|\,² \ge t]$

Proof. Consider such any r. v. $²$, $² \in \{0,1,...\}$ that $\{² \le t\}$ does not dependent on values $¬_0,...,¬_{t-1}$ and $P[² = t] = d_t - d_{t+1}$, $t = 0,1,2,...$. Clearly, $P[² \ge 0] = d_0 - d_1 + d_1 - d_2 + ... = d_0 = 1$, $P[² \ge t] = d_t$, and

$$\sum_{t=0}^{\infty} d_t V_t = \sum_{t=0}^{\infty} P[² \ge t] V_t \ .$$

Let now $f_t := \sum_{k=0}^{t} ¬_k$. From the rearrangement known as the Kolmogorov-Prohorov's theorem it follows that

$$\begin{aligned} Ef_² &= \sum_{t=0}^{\infty} E[f_t ; ² = t] = \sum_{t=0}^{\infty} \sum_{k=0}^{t} E[¬_k ; ² = t] = \\ &= \sum_{k=0}^{\infty} E[¬_k ; ² \ge k] = \sum_{k=0}^{\infty} P[² \ge k] V_k , \end{aligned}$$

where $V_k = E[v_k \mid ² \ge k]$ and $E[¬_t ; A]$, denotes unconditional expectation $E[¬_t I_A]$, I_A is the indicator function of event A. The last assertion follows from the identity $\{² \ge t\} = \{² > t-1\}$, i.e., from the independence of $\{² \ge t\}$ on $±_{t,\infty}$. The change in the order of sums is possible due to the uniform boundness of $E \mid v_t \mid$.

Corollary (Wald's identity). If $¬_0, ¬_1,...$ are independent r.v. or $\{² \ge t\}$, $t = 0,1,2,...$, does not depend on $¬_0, v_1,..., v_{t-1}$, then V_t in both cases of Proposition 2 is unconditional expectation $V_t = \mathrm{E}¬_t$. If $¬_0, ¬_1,...$ are independent identically distributed r.v., then the important Wald's identity follows from Proposition 2:

$$E\sum_{t=0}^{²} ¬_t = \mathrm{E}¬_0 E² \ .$$

Proof: It follows from the following rearrangements:

$$\sum_{t=0}^{\infty} P[² \ge t] = \sum_{k=0}^{\infty} \sum_{t=k}^{\infty} P[² = t] = \sum_{t=0}^{\infty} tP[² = t] = E² \ .$$

Wald's identity is often used implicitly in applications without justification of underlying assumptions that may cause misleading conclusions.

Example 3 (Sets of catastrophic events). A set of stopping time events induces in general a time varying discount rates. Let us show that even a set of geometrically distributed events induces discounting with time declining discount rates. Therefore, the use of constant discount rates for integrated catastrophic risk management may be misleading. Let us assume that there is a set of mutually exclusive events of "magnitude" $i = 1,...,n$. The probability of scenario i is $¯_i$, $\sum_{i=1}^{n} ¯_i = 1$ and, conditional on this scenario, the event i occurs for the first time at $²_i$ with the probability $P[²_i = t] = p_i q_i^t$, $q_i = 1 - p_i$, $t = 0,1,...$. Thus, the occurrence of an event at t is characterized by a mixed geometric distribution $\sum_{i=1}^{n} ¯_i p_i q_i^t$. Let $²$ be the arrival time of a first event. Then $d_t = P[² \ge t] = \sum_{i=1}^{n} ¯_i P[²_i \ge t]$. Since $P(²_i \ge t) = p_i q_i^t + p_i q_i^{t+1} + ... = q_i^t$, then evaluation (3) takes the form

$$V = \sum_{t=0}^{\infty} d_t V_t \ , \ d_t = \sum_{i=1}^{n} ¯_i q_i^t \ . \tag{4}$$

Then $d_t = q_{i^*}^t \sum_{i=1}^{n} ¯_i ¡_i(t)$, where $¡_i(t) = (q_i / q_{i^*})^t$. From $p_{i^•} < p_i$, $p_i = 1 - q_i$, it follows that $¡_i(t) \to 0$, $t \to \infty$, for $i \ne i^•$ and $¡_{i^•}(t) = 1$. Hence, $d_t / q_{i^*}^t \to ¯_{i^*}$ for $t \to \infty$.

Remark 3 (Finite time horizon T). Propositions 1 and 2 hold true also for a finite time horizon $T < \infty$ after substituting probabilities $P[² = t]$ and $P[² \ge t]$ by conditional probabilities $P[² = t \mid ² \le T]$ and $P[² \ge t \mid ² \le T]$.

Remark 4 (On using induced discounting). Propositions 1, 2 provide two alternative approaches for discounting: standard deterministic discounted criterion with an exogenous discounting, or undiscounted random criterion with $²$ defined by arrival time of catastrophic events. Example 4 shows that the corresponding induced discounting $d_t = P[² \ge t]$ can be a complex implicit function of spatio-temporal patterns of these events. It also illustrates that τ and, hence, d_t may depend also on various decisions. All these make it rather difficult to evaluate induced discount factors d_t analytically. Therefore, this obstacle as next section demonstrates requires the use of random stopping time criterion and stochastic optimization methods rather then the standard discounted criterion and deterministic optimization methods. In fact, this approach is similar to the natural selection mechanisms outlined in Example 2. In practice, the approach relies on fast Monte Carlo simulations (Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000; Ermolieva et al., 2003).

CATASTROPHE MODELING: ENDOGENOUS DISCOUNTING

This section summarizes typical motivations for developing spatio-temporal catastrophe models based on Monte Carlo simulators of potential catastrophic events defining random stopping time τ scenarios. A typical model may often include the following loop and the potential for positive feedbacks, branching path-dependencies and disequilibrium calling for proper long-term policy assistance ensuring proper catastrophic risk management:

1. Explicit modeling of stopping time induces discounting in the form of dynamic risk profiles $d_t = P[\tau \geq t]$;

2. The discounting affects optimal mitigation efforts; and

3. Mitigation efforts affect the stopping time τ, risk profiles $P[\tau \geq t]$ and the discounting d_t (return to point 1).

This means that the stopping time criterion induces spatio-temporal endogenous discounting. Consider an important example from studies in (Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000.

Example 4 (Evaluation of a Flood Management Program). Consider a simple version of the catastrophic flood management model developed for the Upper Tisza river region (see discussion in (Ermolieva and Ermoliev, 2005). Throughout the world, the losses from floods and other natural disasters are mainly absorbed by the immediate victims and their governments (Froot, 1997). With increasing losses from floods, governments are concerned with escalating costs for flood prevention, flood response, compensation to victims, and public infrastructure repair. As a new policy, many officials would like to increase the responsibility of individuals and local governments for flood risks and losses, but this is possible only through location-specific analysis of risk exposures and potential losses, the mutual interdependencies of these losses, and the sensitivities of the losses to new risk management strategies.

This is a methodologically challenging task requiring at least the development of spatio-temporal catastrophe models (Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000; Walker, 1997). Although rich data usually exist on aggregate levels, the sufficient location specific data are not available, especially data relevant to new policies. Moreover, catastrophes affect large territories and communities producing mutually dependent losses with analytically intractable multidimensional probability distributions dependent also on various decisions.

As a substitute of real observations, the so-called catastrophe modeling (catastrophe generators) is becoming increasingly important for estimating spatio-temporal hazard exposures and potential catastrophic impacts. The designing of a catastrophe model is a multidisciplinary task. To characterize "unknown" catastrophic risks, that is, risks with the lack of repetitive real observations we should at least characterize the random patters of possible disasters, their geographical locations, and their timing. We should also design a map of values and characterize the vulnerability of buildings, constructions, infrastructure, and activities. The resulting catastrophe model allows deriving histograms of mutually dependent losses for a single location, a particular zone, a country, or worldwide from fast Monte-Carlo simulations rather than real observations (Ermolieva and Ermoliev, 2005; Walker, 1997).

In general, a catastrophe model represents the study region by grids. For example, a relatively small pilot Upper Tisza region is represented by 1500x1500 grids (Ermolieva and Ermoliev, 2005). Depending on the purpose of the study, these grids are aggregated into a much smaller number of cells (locations, compartments) $j = 1,2,...,m$. These cells may correspond to a collection of households at a certain site, a collection of grids with similar land-use characteristics, or an administrative district or grid with a segment of gas pipeline. The choice of cells provides a desirable representation of losses. Accordingly, cells are characterized by their content, in general, not necessarily in monetary units. Values can be measured in real terms, without using an aggregate dollar value. The content of cells is characterized by the vulnerability curves calculating random damages to crops, buildings, infrastructure, etc., under a simulated catastrophic scenario.

Flood occurrences in the Upper Tisza region (Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000; Ermolieva et al., 2003) are modeled according to specified probabilistic scenarios of catastrophic rainfalls and the reliability of dikes. For example, there are nine dikes allocated along the case study region's river branch. Each of them may break after the occurrence at a random time of a 100-, 150-, 500-, and 1000- year rainfall characterized by the up-stream discharge curves calculating the amount of discharged water to the river branch per unit of time. In fact, the discharge curves upscale the information about complex rainfall and run-off processes affected by land-use and land-transformation policies. This brings considerable uncertainty in the definition of a 1/p - year flood, $p = 1/100$, 1/150, 1/500, 1/1000. For the simplicity of notation, let us consider only $p = 1/100$. Therefore, a 100-year discharge curve may represent, in fact, a set of mutually exclusive floods characterized by a finite number of probabilistic scenarios as in Example 3.

The stopping time can be defined differently, depending on the purpose of the policy analysis. The time ² can be defined by the first arrival time of rains within a given set of discharge curves. In this studies $d_t = P[\text{²} \ge t]$ depends only on t. A catastrophic flood in our example occurs due to the break of a dike. These events are considered as mutually exclusive events because the break of a dike in the pilot region releases the "pressure" on other dikes. Therefore, the stopping time ² can be defined as the first time moment of a dike break. In this case, the probability or induced discount factor $d_t = P[\text{²} \ge t]$ is an implicit function of t, probabilities of discharges, the probability of a dike break, maintenance of the flood protection system, e.g., modifications to the dikes, the removal of some of them, building new retention areas and reservoirs. Besides these structural decisions, the stopping time ² may be affected by other decisions, e.g., land use policies. Accordingly, depending on goals, the definition of stopping time ² can be further modified. For example, let us assume that the region (Ermolieva et al., 2003) participates in the flood management program through payments to a mutual catastrophe fund, which has to support a flood protection system including the reliability of dikes and compensates losses to victims. To enforce the participation in the program, the government provides only partial coverages of losses. The stability of this program critically depends on the insolvency of the fund that may require a new definition of ². Let   be a fixed investment rate enabling the support of the system of dikes on a certain safety level and ¶ be a random time of a first catastrophic flood. Denote by $L_j^{\text{¶}}$ random losses at location j, $j = \overline{1,m}$, at time $t = \text{¶}$ and by ®_j the premium rate paid by location j to the mutual catastrophe fund. Then, its accumulated risk reserve at time ¶ together with a fixed partial compensation of losses $\text{¡} \sum_j L_j^{\text{¶}}$ by the government is $R_{\text{¶}} = \text{¶} \sum_j \text{®}_j + \text{¡} \sum_j L_j^{\text{¶}} - \sum_j \text{¨}_j L_j^{\text{¶}} - \ \text{¶}$, where $0 \le \text{¨}_j \le 1$, is the portion of losses compensated by the fund at location j. Let us also assume that the functioning of the flood management program is considered as a long-term activity assuming that growth and aging processes compensate each other. Then, the insolvency of the fund is associated with the event:

$$\text{¶} \sum_j \text{®}_j + \text{¡} \sum_j L_j^{\text{¶}} - \sum_j \text{¨}_j L_j^{\text{¶}} - \ \text{¶} < 0 \,. \tag{5}$$

The sustainability also depends on the willingness of individuals to accept premiums, i.e., on the probability of overpayments:

$$\text{²®}_j - \text{¨}_j L_j^{\text{²}} > 0 \,, \ j = 1,...,m \,.$$

Inequality Eq.5 defines extreme random events affected by various feasible decisions x including components $(\text{®}_j, \text{¨}_j, \text{¡}, b_j, \ , j = \overline{1,m})$.

The likelihood of event Eq.5 determines the resilience of the program. It can be expressed by the probabilistic constraint:

$$P\left[\left(\text{²} \sum_j \text{®}_j + \text{¡} \sum_j L_j^{\text{²}} - \sum_j \text{¨}_j L_j^{\text{²}} - I\text{²} < 0\right)\right] \le \text{¥} \,, \tag{6}$$

where ¥ is a specified "survival" level requiring, say, that a "collapse" may occur only once in 10^7 years, $\text{¥} = 10^{-7}$.

It is more natural now to define the stopping time ² as the first time when event Eq.5 occurs. In this case ² would depend on components of vector x and the induced stopping time discounting would focus on time horizons associated with the occurrence of the event Eq.5 characterizing the resilience of the flood risk management program. Detailed analysis and solution procedures of more general cases can be found in (Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000; Ermolieva et al., 2003.

Remark 5 (Dynamic risk profiles and CVaR risk measure). The following Example 5 and Remark 2 illustrate that the probability distributions $P[\text{²} \ge t]$, $t = 0,1,...$, itself represent key safety characteristics of catastrophic risk management programs. Induced discounting $d_t = P[\text{²} \ge t]$ then "controls" these risk profiles implicitly through their contributions to discounted goals of programs. This also allows to impose explicitly safety constraints of the type $P[\text{²} \ge t] \ge \text{¥}_t$ for some safety levels ¥_t, $t = 0,1,...$ (see e.g.

Ermoliev et al., 2003; O’Neill et al., 2006). In this minimizing stopping criteria robust strategies would indirectly control the safety constraints.

Example 5 (Vital thresholds). The occurrence of disasters is often associated with the likelihood of some processes abruptly passing “vital” thresholds. This is a typical situation for insurance, where the risk process is defined by flows of premiums and claims whereas thresholds are defined by insolvency constraints of type Eq.5. A similar situation arises in the control of environmental targets and in the design of disaster management programs (Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000). Assume that there is a random process R_t and the threshold is defined by a random $°_t$. In catastrophe modeling, R_t and $°_t$ can be large-dimensional vectors reflecting the overall situation in different locations of a region. Let us define the stopping time ² as the first time moment t when R_t is below $°_t$. By introducing appropriate risk management decisions x it is often possible to affect R_t and $°_t$ in order to ensure the safety constraints $P[R_t \geq °_t] \geq ¥$, for some safety level ¥, $t = 0,1,2,...$.

The use of this type of safety constraints is a rather standard approach for coping with risks in the insurance, finance, and nuclear industries. For example, the safety regulations of nuclear plants assume that the violation of safety constraints may occur only once in 10^7 years, i.e. $¥ = 1 - 10^{-7}$. It is remarkable that the use of stopping time criterion as in the right-hand side of Eq.2 has strong connections with the dynamic safety constraints and dynamic versions (Ermoliev et al., 2003; O’Neill et al., 2006) of static CVaR risk measures (Rockafellar and Uryasev, 2000), as Remarks 2 and 5 point out.

INTERTEMPORAL INCONSISTENCY PROVOKING CATASTROPHES

The time consistency of discounting means that the evaluation of an investment project today ($t = 0$) will have the same discount factor as the evaluation of the same project after any time interval $[0,T]$ in the future. In other words, despite delayed implementation of the project we always found ourselves in the same environment. Only standard geometric or exponential discounting with constant discount rates $d_t = d^t = e^{(\ln d)t} = e^{-ªt}$, $ª = -\ln d$, define a homogeneous time consistent preference:

$$\sum_{t=0}^{\infty} d^t V_t = V_0 + dV_1 + ... + d^{T-1}V_{T-1} + d^T[V_T + dV_{T+1} + ...].$$

This is also connected with the well-known “memoryless” feature of geometric and exponential probability distributions: if $P[² \geq t] = d^t$, $0 < d < 1$, then for any $t \geq 0$, $s \geq 0$,

$$P[² = t + s \,|\, ² \geq t] = d^{t+s}(1-d)/d^t = (1-d)d^s.$$

Hence, independently of waiting time t, the probability of the stopping time occurrence at $t+s$ is the same as at the initial time moment $t = 0$.

For other discount factors with time-dependent rates, their time inconsistency requires appropriate adjustments for projects undertaken later rather than earlier, e.g. due to aging process, say, of dikes. The misperception of this inconsistency may provoke increasing vulnerability and catastrophic losses (Cline, 1999). Let us consider typical scenarios of such developments.

A number of authors distinguish between various types of so-called “imperfect altruism” (see e.g. Grollman and Simon, 2003; Winkler 2006) resulting in the lack of social commitment to mitigate risks. For example, there exist definitions of a naïve, a sophisticated and a committed (ideal) society. The main differences between these three societies and how they provoke catastrophes are summarized in (Ermolieva et al., 2003; Winkler 2006) by using a simplified flood management model outlined in previous section.
The region is subdivided into 20 aggregated subregions (locations). Random observations of dike breaks and location specific losses are simulated by a specific inundation model. The governmental compensation ¡, location-specific vulnerabilities of constructions and values have been modified from original data (Project Proposal, 1997) in order to ensure “survival” constraint Eq.6 for $¥ = 0.01$.
We assume that the dike system deteriorates over time, which is represented by the probability of a dike j break with time-dependent deterioration rate of the form $1/(1+¢_{jt})$, $¢_{jt} = \sim_j ¢^t$, $¢ < 1$, $\sim_j > 0$.
We consider the fixed 100-year horizon (see Table 1) in which five generations of three societies, naïve, sophisticated, and committed (discussed in Ramsey 1928), live and plan how to cope with catastrophic losses if they occur. The societies are able to mitigate the risks by accumulating money in a catastrophe fund to be able to support dike maintenance and compensation of losses. But, depending on their perception of risks and time inconsistency, the results are different. It is assumed that the rate of premiums into the fund is

fixed on the level necessary to compensate the deterioration rate of dikes. Details of the solution procedure can be found in (Ermolieva, 1997; Ermolieva and Ermoliev, 2005; Ermoliev et al., 2000) for conceptually similar mathematical models. The following provides only a summary of the numerical analysis.

The Naïve Society. We assume that within 100 years each of five generations plans for 20 years. The current generation of social planners is aware of possible catastrophes. It plans catastrophe management, similar as in the model of previous section, by establishing a catastrophe fund and paying premiums. Unfortunately, the society postpones the implementation of decisions because it puts first priority on current consumption. It has a misleading view on catastrophes. Namely, if a catastrophe has not occurred in the later generation the society believes that with the same probability it will not occur within the current generation. The society fails to take into account the time inconsistency induced by increasing probability of a dike break without proper maintenance.

For the next generation the time is shifted forward by 20 years, and the second generation, similar to the first, plans but does not implement saving actions essential for a catastrophe fund to function. In the same way we simulate the other three generations, each time calculating how much premiums they need to lay aside. The plans are never implemented and the view on a catastrophe is time invariant consistent with the initial geometric probability distribution.

Table 1. Performance of societies.

Society	Planning period	Ruin probability required	Premium	Ruin probability actual
Naïve	0 - 100	0.01	1.62	0.32
	20 - 100	0.01	1.32	0.46
	40 - 100	0.01	0.97	0.61
	60 - 100	0.01	0.63	0.76
	80 - 100	0.01	0.35	0.89
Sophisticated	0 - 100	0.01	1.92	0.32
	20 - 100	0.01	2.49	0.46
	40 - 100	0.01	3.06	0.61
	60 - 100	0.01	3.63	0.76
	80 - 100	0.01	4.19	0.89
Committed	0 - 100	0.01	2.1	0.01

What may happen to five generations of the *Naïve* society is shown in Table 1. Since plans including dike maintenance are not implemented, the "Ruin probability actual" deviates from the initially specified value ("Ruin probability required"). Yet, the society believes that the ruin probability satisfies the desirable level 0.01. Therefore, it even decreases the premiums in the remaining intervals ("Premium per location"). In reality, the actual threat of dike break increases ("Ruin probability actual") due to dikes deterioration.

The Sophisticated Society. The planning procedure for the sophisticated society is similar to that of the naïve society (Table 1). In contrast to naïve, this society correctly understands the time inconsistencies induced by the deteriorating system of dikes and the need for their adequate maintenance. However, similar to the naïve planners, it also evaluates the present consumption much higher, i.e., spends more than what is allowed by the risk management plan.

Thus, the sophisticated society understands the need to increase premiums for proper dike maintenance, but this decision is postponed from one generation to another. Due to these delays, the risk burden is increasingly shifted to next generations ("Ruin probability actual"), i.e. the premiums are not accumulated, dikes are not maintained and proper compensation of losses is impossible. The "pathologies" of the societies can be explained by their ignorance of risks, incorrect understanding of time inconsistency and the lack of committed intergenerational actions.

The Committed Society. The committed society evaluates risk management strategies by taking into account time dependent profiles of catastrophic risks and induced discounting. This society is able to implement decisions for all subsequent generations. Table 1 shows the premiums that the society pays are much lower than those of the sophisticated, which is a direct consequence of their committed actions.

CONCLUDING REMARKS

The proposed new approach to discounting is based on undiscounted stopping-time criterion. Discount factors d_t , $t = 0,1,...$ can be associated with the occurrence of an extreme "stopping time" ("killing") event at random time τ with probability $P[\tau \geq t] = d_t$. Consequently, the infinite discounted sum $\sum_{t=0}^{\infty} d_t V_t$, $V_t = E\neg_t$, is replaced by the undiscounted expectation $E\sum_{t=0}^{\tau} v_t$ within the finite interval $[0, \tau]$. The use of the stopping time criterion $E\sum_{t=0}^{\tau} v_t$ induces the standard discounting in the case when τ is associated with the lifetime of market products. For catastrophic risks, the stopping time τ can be associated with the arrival time of potential catastrophic events. Apart from focusing on catastrophic events, the criterion $\sum_{t=0}^{\tau} v_t$ allows also to address the variability of valuations even in the case of deterministic flows V_0 , V_1 , …. In this case, it is often important to substitute the expected value of random sum $\sum_{t=0}^{\tau} v_t$ by its quantiles. Existing stochastic optimization (STO) methods allow to do this without destroying convexities. Mitigation efforts affect the occurrence of extreme events and, thus, they affect discounting, which in turn affects mitigations. This endogeneity of discounting restricts exact evaluations of d_t and the consequent use of deterministic methods and it calls for stochastic optimization approaches.

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