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HOW TO MEASURE THE IMPORTANCE OF CLIMATE RISK FOR DETERMINING OPTIMAL GLOBAL ABATEMENT POLICIES?

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We investigate the importance of explicitly accounting for uncertainty in the determination of optimal global climate policy. We demonstrate that the marginal risk premium determines the importance of adapting the optimal policy to uncertainty. Common integrated assessment models (IAM) of climate change suggest uncertainty has little effect because the marginal risk premium in these models is small. A rigorous investigation of the marginal risk premium and the marginal functional relationships within IAMs allows understanding the non-significance of (thin-tailed) uncertainty as a result of compensating factors in the climate cause-effect chain.

Keywords: Integrated assessment; climate change; uncertainty; climate risk; mitigation policies.

1. Introduction

Global Climate Change Analysis is surrounded by large uncertainties about key parameters in the socio-economic system and the climate system. The uncertainties arise from imperfect knowledge about the dynamics of the subsystems, from internal short term dynamics or stochasticity and from the long time lag between cause and effect within the climate system (Tol, 1999). Analysts use models that integrate the socio-economic system and the climate system to determine welfare-optimal, long-term strategies to mitigate climate change (Nordhaus, 1994). A key question concerning uncertainty and climate change assessment is whether the analysts should explicitly

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account for uncertainty in their integrated assessment models to capture the effect of uncertainty on decisions?

Introducing uncertainty about the climate response to greenhouse gas emissions and resulting climate damages has two effects: Firstly, assuming a right-skewed damage distribution and some degree of risk aversion, the introduction of potential high-damage cases adds a risk-premium to all mitigation efforts. The risk premium increases the net-benefit of the optimal best-guess climate policy, but as long as the risk premium itself does not depend on the level of mitigation effort, the optimal mitigation policy does not change due to the introduction of uncertainty. Thus, secondly, the optimal climate policy changes, if and only if uncertainty also introduces a non-zero marginal risk premium. Hence, the question for the significance of adapting the optimal mitigation policy to uncertainty can be answered by comparing the welfare benefit of adapting the optimal climate policy to the overall benefit of acting upon climate change.

Over the last two decades following the pioneering work of Nordhaus (1994), many contributions have been made to answer the question how explicitly accounting for uncertainty changes the optimal climate policy (e.g., Heal and Kriström, 2002, and references therein; Keller *et al.*, 2004; Wirl, 2007). Some studies (e.g., Pizer, 1997) also investigated the welfare effect from introducing uncertainty. A common result has emerged from cost-benefit analyses with integrated assessment models that optimize the trade-off between mitigation costs of reducing greenhouse gas emissions and climate change induced damage costs (e.g., Nas, 1996). Although the optimal climate policy might change significantly due to the introduction of uncertainty, the welfare gain associated with adjusting the policy to uncertainty is negligible (Pizer, 1997).

This counter intuitive result has led to a number of investigations of changes to the structure of the integrated assessment models that would provide strong arguments for stricter mitigation under uncertainty (for an overview of methods and changes, see e.g., Kousky *et al.*, 2011).

Weitzman (2009) shows that the effect of uncertainty can become significant, and even dominating, if fat-tailed climate response risk and exponential climate damages are considered. Under certain conditions, such fat-tailed climate damage can lead to unbounded expected welfare losses. However, this theoretical result does not answer the main question from above satisfactorily. Even if a very large, and potentially infinite, risk premium is added to the optimal best-guess policy, the adjustment of optimal policy to uncertainty is only important if some mitigation policy can be found that significantly reduces the fat-tail risk, i.e., the question of the size of the marginal risk premium remains to be answered. Weitzman (2010) emphasizes this point by stating that the choice of optimal climate policy might be determined by a very arbitrary calibration of the low-probability but high impact end of the damage function and probability distributions. However, he does not provide a systematic analysis of the conditions under which the welfare benefit from adjusting optimal climate policy to uncertainty becomes a significant part of the overall welfare benefit of acting upon climate change.

Others have also included some representation of climate catastrophy into integrated assessment models (e.g., Ackerman *et al.*, 2010; Gerst *et al.*, 2010) and have shown that the inclusion of catastrophic risk can favor more stringent abatement scenarios, however they have not explicitly investigated the question of the significance of the marginal risk premium between the optimal best guess and the optimal policy under uncertainty, rather they investigate the changes in benefits due to uncertainty for a small set of specific mitigation policies. Newbold and Daigneault (2009) investigate the marginal risk premium of the willingness to pay of a decision maker to prevent severe climate change within a simple two period analytic model. Here again they find, that uncertainty leads to stronger optimal mitigation efforts. But when transferring the result to a more complex integrated assessment model they go back from analyzing optimal policies to only analyzing two example emission scenarios and thus are not able to determine the importance of explicitly including uncertainty for the determination of optimal emission policies.

Other explanations put forward as reason for negligible effects from uncertainty on optimal mitigation decisions (and resulting welfare gains) are the limitations of the standard expected utility framework in representing the decision maker's preferences towards uncertainty. Within the standard expected utility framework the parameter representing the decision makers aversion agains risk (RRA) is equal to the inverse of the elasticity of intertemporal substitution of consumption (EIS). As both parameters have been shown to affect the optimal decision under uncertainty in different directions, fixing both parameters to the same value might underestimate the effect from including uncertainty. This limitation is known in the theoretical literature on decision theory since the 1970's, and a number of alternative preference representations have been proposed that would allow for a decoupling of the relative risk aversion and EIS parameters (e.g., Kreps and Porteus, 1978; Traeger, 2009). The former representation has been implemented in the climate mitigation context (e.g., by Ha-Duong and Treich, 2004; Kaufman, 2012), and it has been shown that the decoupling of both parameters indeed leads to a far higher risk premium of mitigation strategies. However, these applications do face two problems: First, again they investigate the risk premium for a set of fixed mitigation strategies and do not compare the optimal best-guess and optimal strategy under uncertainty. Second, and more important, it has been shown by Traeger (2009, and references therein) that the recursive preference representation by Kreps and Porteus is flawed in that it introduces an implicit preference of the decision maker for the timing of uncertainty resolution, a feature that arguably is normatively unappealing. The improved framework of Traeger, in which this problem is potentially resolved, still awaits an independent confirmation and an application within the climate context.

Another feature not represented by the expected utility framework is the empirically well established aversion of real world decision makers against a situation in which the probability distribution describing the risk in a variable is itself uncertain (see e.g., Woodward and Bishop, 1997). This situation is termed ambiguity. Lange and Treich

(2008) have shown that including ambiguity aversion into an integrated assessment model again can favor stricter abatement under uncertainty.

Finally, the expected utility theory assumes a single, representative agent who decides on behalf of an underlying population. This assumption implies equality amongst all individuals, or groups of individuals both in terms of their level of welfare without climate change and in terms of the impacts they are facing from climate change. Both assumptions are rather strong abstractions, and reintroducing inhomogeneity into the framework can lead to far stronger effects of uncertainty on optimal policy levels (Schmidt *et al.*, in press).

But even leaving all these structural changes aside, which do indeed lead to a significant effect of uncertainty on optimal decisions and resulting welfare gains, the question remains why there is such a small effect of uncertainty in the standard integrated assessment models of climate policy? As pointed out in the theoretic literature, uncertainty has no effect on decisions and welfare if the model is (nearly) linear in the uncertain parameters (see e.g., Baker, 2006; Lange and Treich, 2008). Furthermore, the marginal utility of the model also needs to be non-linear in the uncertain parameter for the optimal decisions to be effected, as they are determined by the trade-off between marginal benefits and costs. However, these theoretical conditions do not explain why uncertainty has nearly no effect on welfare within the more complex integrated assessment models, as those models do incorporate a number of (partly strong) nonlinearities: The welfare function, the temperature response to carbon emissions, the damage function etc.

This study builds on the existing literature and provides two important new components: First, we present a diagnostic for analysing the importance of explicitly including uncertainty into complex integrated assessment models. The diagnostic is based on and tested for the integrated assessment model MIND. The general applicability to all integrated assessment models rests on the validity of a projection of their multi-dimensional decision spaces onto a scalar decision variable and remains to be shown. Second, by applying this diagnostic to the integrated assessment model MIND, we give an explanation of the negligible uncertainty effect in the standard setting not from the absense of structural changes but from an understanding of the marginal functional structure within the climate cause-effect chain in the model.

More concretely, we introduce a decomposition of the overall benefit of climate policy into single components: The benefit of the best guess policy, the re-evaluation of the best guess policy under uncertainty which adds a risk premium to the optimal best guess policy, and the value of adapting the optimal policy to uncertainty which is equivalent to the differential risk premium (or marginal risk premium) between the optimal policy under uncertainty and the optimal best guess policy (evaluated under uncertainty). We apply the decomposition to the integrated assessment model MIND (Edenhofer *et al.*, 2005) and analyze the relative importance of all components. Additionally we project the complex integrated assessment model to an a-temporal marginal cost-benefit picture. This allows us to connect the different components of the overall benefit of climate policy to the functional form of the marginal benefits and costs.

Applying this methodology to the integrated assessment model MIND we can identify two main reasons for the negligible effect of accounting for uncertainty in the model: First, the uncertainty effect expected from the nonlinear damage function (convex increasing in cumulated emissions) is compensated by the saturation effect of temperature increase as response to increasing cumulated emissions (concave in cumulated emissions). This limits both the overall benefit of climate policy, and the welfare effect from adapting optimal policy to uncertainty.

Second, the strongly convex increasing marginal mitigation costs lead to an optimal best-guess policy that already lies within a regime of relative high mitigation costs. Thus, a moderate change of (expected) marginal benefits due to the introduction of thin-tailed uncertainty cannot change the optimal policy much.

Using this understanding of the relationship between the model formulation, the shape of the marginal benefits and costs, and the components of the benefit of climate policy, we introduce several changes in the model structure to create a significant welfare effect from uncertainty. These changes include a sensitivity analysis with respect to the parameter of constant relative risk aversion, a switch towards exponential damage and an implementation of a linear climate response to cumulative carbon emissions as proposed by Matthews *et al.* (2009).

The paper is structured as follows: Section 2 introduces the general climate decision problem under uncertainty and describes the decomposition of the overall benefit of climate policy into the single components determining the benefit of climate policy under uncertainty. In Sec. 3, the framework is applied to the Model of Investment and Technological Development (MIND). The importance of introducing uncertainty is compared to the overall benefit of acting against climate change. The sensitivity of the results towards changes in normative parameters is evaluated. The marginal cost-benefit picture of the model as well as the functional dependency along the climate cause-effect chain is presented to investigate the origin of the negligible welfare effect from uncertainty. In Sec. 4, several changes to the model structure are investigated with respect to their influence on the importance of uncertainty: Changes in the parameter of constant relative risk aversion, exponential damage, and linear climate carbon response. Section 5 concludes.

2. How to Measure the Importance of Uncertainty?

2.1. *The decision problem*

First, we formulate the general decision problem incorporating uncertainty in its simplest version. We only consider one decision period. The principle agent (DM) decides upon a set of decision variables x, like investments, emission control rates, etc.

The decisions might also represent a whole time path of single decisions $x(t)$. Depending upon the decisions and upon the state of the world (SOW) the DM derives an overall welfare $U(x,\theta)$. Uncertainty about the SOW is represented by a probability distribution $\pi(\theta)$.

Technically speaking, we model an open loop optimal control problem and thus neglect the effect of changes in available information over time. The problem now is to maximize the overall expected utility $V(x,\pi)$:

$$\max_x V(x,\pi) = \max_x \sum_j \pi(\theta_j)U(x,\theta_j). \tag{1}$$

Second, we introduce the cases of the DM's information structure relevant for the climate change example. The random variable θ represents the uncertain magnitude of the temperature response to greenhouse gas emissions and of climate change induced damage. The DM's knowledge, or belief, about the values of the uncertain climate response and damage is represented by the probability distribution function $\pi(\theta)$. The general case of uncertainty is simply denoted by π. The degenerate case, where the DM is certain about θ taking the value θ_j, is defined via:

$$\pi_j \equiv \pi\begin{cases} 1 & \theta = \theta_j \\ 0 & \text{else} \end{cases}. \tag{2}$$

Two special cases of the degenerate distribution are the case of no climate damage at all, denoted by π_0, and the case of certainty about θ taking its expected value, denoted by $\bar{\pi}$:

$$\pi_0 \equiv \pi\begin{cases} 1 & \theta = 0 \\ 0 & \text{else} \end{cases}, \quad \bar{\pi} \equiv \pi\begin{cases} 1 & \theta = \sum_j \pi(\theta_j)\theta_j \\ 0 & \text{else} \end{cases}. \tag{3}$$

Third, utilizing the decision framework and the special instances of information structure, we define the following three policy scenarios, relevant for measuring the importance of uncertainty:

The No-Control Case (NC): We define the policy case of NC as the optimal policy in the absence of any climate damage: $\hat{x}_0 \equiv \arg\max_x V(x,\pi_0)$. The rationale behind this definition stems from the difference between the non-cooperative and the cooperative solution of a decentralized market economy. In such an economy, a no-control behavior is caused by the imperfect cooperation of a large number of decision makers. In a competitive setting, each decision maker only anticipates her own small share of global climate induced economic damage leading to an almost total neglect of the climate problem in individual decisions. Thus, the climate problem is called an externality to the market. In a fully cooperative setting the single actors would optimize their combined welfare and thus correctly anticipate global warming. Within the model, we simulate the lack of cooperation by making the DM ignorant towards climate change.[1]

The Best-Guess Case: We define the optimal policy under certainty about climate response and damage by $\hat{x}_1 \equiv \arg\max_x V(x, \bar{\pi})$. This is the common approach to take the expected value of the uncertain parameters as best guess values.

The Uncertainty Case: We define the optimal decision under uncertainty about climate response and damage by $\hat{x}_2 \equiv \arg\max_x V(x, \pi)$.

2.2. *Metrics for measuring the importance of uncertainty*

In this section, we use the nomenclature defined above to introduce metrics that measure the different components of the overall benefit of climate policy separately. Combining the above defined policy scenarios $(\hat{x}_0, \hat{x}_1, \hat{x}_2)$ with the possible assumptions of how the world reacts to the policy decisions, represented by information structures $(\pi_0, \bar{\pi}, \pi)$, leads to 9 possible outcomes in terms of expected utility V. The combinations relevant for the remainder of the discussion are depicted schematically in Fig. 1. The welfare differences between those cases, measured as changes in certainty and balanced growth equivalents (ΔCBGE, see Appendix A), can be used as metrics for the importance of the different effects of uncertainty in welfare terms:

The relevant measure for the importance of climate policy in a best-guess world is the net benefit of reacting to climate change, i.e., changing from $\hat{x}_0$ to $\hat{x}_1$, $\mathrm{BCP}(\hat{x}_1, \bar{\pi}) = \Delta\mathrm{CBGE}(V(\hat{x}_1, \bar{\pi}), V(\hat{x}_0, \bar{\pi}))$. This benefit of climate policy is small compared to the mitigation costs within a cost-efficiency framework if the mitigated damages are roughly the same size as the mitigation costs. This is the case within the MIND model. As will be shown later, the benefit of climate policy can become large, compared to the mitigation costs, if the marginal benefit of mitigation is strongly convex in the level of abatement.

Introducing uncertainty has two effects: First, the valuation of policies $\hat{x}_0$ and $\hat{x}_1$ changes. If, for all x, the (expected) utility $V(x, \bar{\pi})$ is concave (convex) in the uncertain parameter θ, the expected utility for $\hat{x}_0$ and $\hat{x}_1$ in the uncertain case (π) will be smaller (larger) than in the best-guess case ($\bar{\pi}$). As the benefit of climate policy $\mathrm{BCP}(x, \bar{\pi})$ is defined as difference between two levels of (expected) utility the behavior of $\mathrm{BCP}(x, \bar{\pi})$ for a switch between the best-guess and the uncertainty world depends on the curvature of the marginal (expected) utility in the uncertain parameter θ. If, for all x, the marginal (expected) utility is convex (concave) in θ, then the increase (decrease) of (expected) utility due to uncertainty in θ compared to the best guess $\bar{\theta}$ is smaller (larger) for $\hat{x}_1$ than for $\hat{x}_0$. Hence the benefit of adopting the optimal climate policy from the best guess world increases (decreases) when evaluated in the uncertain

[1]Within our setting the no-control case is not only suboptimal due to the lack of mitigation efforts, but additionally the savings rate cannot be adjusted to the observed climate damage. Thus, the benefit from internalizing climate damage is slightly exaggerated. However, this error is small, as the savings rate adjustment due to climate damage only becomes significant for very high levels of climate damage ($D \gg 50\%$).

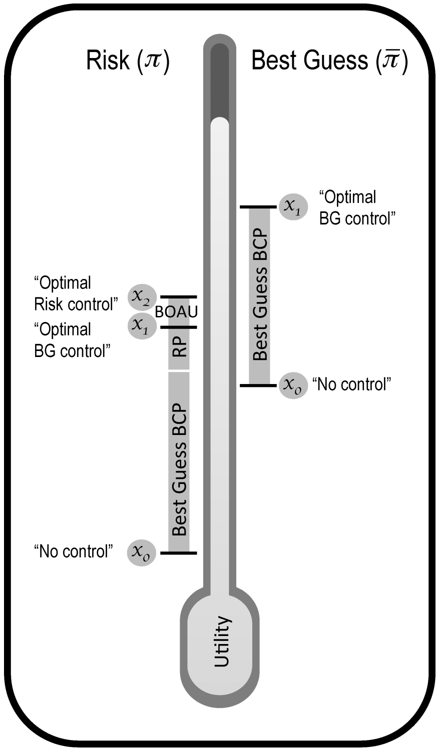

Figure 1. Welfare levels for the combinations of policy scenarios (the no-control case x_0, the optimal climate policy in the best-guess case x_1, and the optimal climate policy under uncertainty x_2) and information structures (the best-guess case $\bar{\pi}$ and the uncertain case π) that are relevant for defining importance metrics. Also shown are relevant welfare differences, measured in ΔCBGE: the benefit of climate policy in the best-guess information setting (Best-Guess BCP), the risk premium *RP* from reevaluating the best-guess climate policy x_1 under uncertainty (π), and the benefit of anticipating uncertainty (BOAU) from adjusting the optimal climate policy under uncertainty from x_1 to x_2.

world: $\mathrm{BCP}(\hat{x}_1, \pi) > (<) \mathrm{BCP}(\hat{x}_1, \bar{\pi})$. The difference between the benefit from the best guess policy in the uncertain world and the certain world is called a risk premium, denoted by $\mathrm{RP}(\hat{x}_1) = \mathrm{BCP}(\hat{x}_1, \pi) - \mathrm{BCP}(\hat{x}_1, \bar{\pi})$.

Second, when not only evaluating the solution of the best guess world under uncertainty but explicitly maximizing the expected utility, the optimal climate policy will change (from $\hat{x}_1$ to $\hat{x}_2$). This possibility of adjusting climate policy to uncertainty leads to an increase in overall expected utility, denoted benefit of adopting to uncertainty,[2] $\mathrm{BOAU}(\hat{x}_1, \hat{x}_2, \pi) \equiv \mathrm{BCP}(\hat{x}_2, \pi) - \mathrm{BCP}(\hat{x}_1, \pi)$.

Taking both effects of uncertainty into account, the overall benefit from optimally responding to climate change under uncertainty $\mathrm{BCP}(\hat{x}_2, \pi)$ can be divided into

[2]The welfare difference actually is the difference between the risk premium for the optimal policy under uncertainty $\hat{x}_2$ and the optimal best guess policy $\hat{x}_1$, which is equivalent to the integral over the marginal risk premium between $\hat{x}_1$ and $\hat{x}_2$.

three parts:

$$\mathrm{BCP}(\hat{x}_2, \pi) \equiv \mathrm{BCP}(\hat{x}_1, \bar{\pi}) + \mathrm{RP}(\hat{x}_1) + \mathrm{BOAU}(\hat{x}_1, \hat{x}_2, \pi). \tag{4}$$

A common measure for the "strength" of this adjustment effect is the absolute or relative change in optimal decisions itself, i.e., $\Delta\hat{x} \equiv (\hat{x}_2 - \hat{x}_1)/\hat{x}_1$ (e.g., see Tol, 1999). We argue that the comparison of optimal policies is insufficient to decide upon the importance of including uncertainty as even a large Δx does not necessarily has to correspond to a large BOAU. To assess the importance of uncertainty and of optimizing expected utility, we compare the single contributions of Eq. (4) normalized to their sum.[3,4]

3. Importance of Uncertainty in MIND

Why does accounting for uncertainty about the climate response and the climate damage change the results in standard applications of integrated assessment models of climate change only to a small degree? In this section, we investigate this question within the integrated assessment model MIND by applying the decomposition of the benefit of climate policy presented above. For this purpose, we introduce uncertainty about climate sensitivity and the severity of climate change induced damage into the MIND model (Sec. 3.1). First, we reproduce the findings in the literature (e.g., Pizer, 1997; Manne, 1995) that explicitly including uncertainty has a small influence on the benefit of climate policy (Sec. 3.2). We then interpret the MIND model in a "cost-benefit" picture and resolve the functional dependencies between the decision variables and the resulting marginal benefits and costs of climate change mitigation to shed light on the structural reasons for a negligible welfare effect from adapting optimal mitigation policy to uncertainty.

[3]As done by Pizer (1997), who uses the relative measure $\mathrm{BOAU}/\mathrm{BCP}(\hat{x}_2, \pi)$ to assess the importance of optimizing under uncertainty.

[4]The formalism can easily be extended to include the welfare effect from optimally adjusting mitigation policy to the arrival of new information. This situation has been (implicitly) investigated, e.g., by (Nordhaus, 2008; Ackerman *et al.*, 2010) by performing a Monte Carlo sensitivity study of the integrated assessment model (and aggregating expected utility over all possible parameter values ex-post). Within this limiting case of early learning, the optimal policy can be chosen conditional on the perfect knowledge about the respective state of the world, $\hat{x}_3 = \hat{x}_3(\theta)$. The expectation for the overall welfare is now not taken over uncertain states of the world, but over the possible messages leading to certain states of the world. To be consistent with the ex ante knowledge, the distribution over the messages has to be identical to the distribution over the SOW in the uncertain case. The benefit of perfect learning is measured by comparing the expected utility with and without learning. The Value of Perfect Information is defined via: $\mathrm{VPI}(\pi) \equiv \mathrm{BCP}(\hat{x}_3(\theta), \pi) - \mathrm{BCP}(\hat{x}_2, \pi)$. The relative importance of perfect learning can be compared to the importance of maximizing under uncertainty and the importance of reevaluating the optimal best guess policy by dividing the overall benefit of acting upon climate change under perfect learning into the components:

$$\mathrm{BCP}(\hat{x}_3(\theta), \pi) \equiv \mathrm{VPI}(\pi) + \mathrm{BCP}(\hat{x}_1, \pi) + \mathrm{RP}(\hat{x}_1) + \mathrm{BOAU}(\hat{x}_1, \hat{x}_2),$$

and then comparing the normalized contributions of the single effects.

3.1. *The model of investment and technological development (MIND)*

We employ the Model of Investment and Technological Development (MIND) (Edenhofer *et al.*, 2005) in its stochastic version presented by Held *et al.* (2009). Additionally we include learning as introduced to the model by Lorenz *et al.* (2011). MIND is a model in the tradition of the Ramsey growth model and similar to the well-known DICE model (Nordhaus, 1993). The version we use differs from the classical Ramsey model in two major aspects: Firstly, the production sector depends explicitly on energy as production factor, that is provided by a crudely resolved energy sector. The energy sector contains (i) fossil fuel extraction, (ii) secondary energy production from fossil fuels, and (iii) renewable energy production. The macroeconomic constant-elasticity-of-substitution (CES) production function depends on labor, capital and energy as input factors. Secondly, technological change is modeled endogenously in two ways. The DM can invest into research and development activities to enhance labor and energy efficiency. Additionally, productivity of renewable and fossil energy producing capital increases with cumulative installed capacities (learning-by-doing). We assume welfare to be an inter-temporally separable isoelastic utility function of per capita consumption with a constant relative risk aversion $\eta = 1.5$ that is changed for the sensitivity study later on. It takes the form:

$$U(c(I,s)) = \sum_{t_0}^{t_e} L(t) \cdot \frac{1}{1-\eta}\left[\left(\frac{[c(I,s)](t)}{L(t)}\right)^{1-\eta} - 1\right] e^{-\rho t} dt, \tag{6}$$

where $I = (I_{\mathrm{K}}, I_{\mathrm{R\&D}}, I_{\mathrm{Fossil}}, I_{\mathrm{Renewables}})$ is the vector of investment flows in the different sectors over time, s is the unknown state of the world, ρ is the pure rate of social time preference taken to be 0.01/yr, and $L(t)$ is an exogenously given population scenario. Investments are related to the global consumption $[c(I,s)](t)$ via the budget constraint:

$$Y_{\mathrm{net}}(t,s) = [c(I,s)](t) + \sum_n I_n(t,s), \quad c(I,s) \geq 0, \tag{7}$$

with the Gross World Product (GWP) Y_{net} net of climate related damage. Y_{net} is related to gross GWP over $Y_{\mathrm{net}} = Y_{\mathrm{gross}} \cdot DF$, where DF is a multiplicative damage factor defined by the damage function (see Roughgarden and Schneider, 1999):

$$DF(T) = \frac{1}{1 + a \cdot T^b}. \tag{8}$$

3.2. *Importance of uncertainty in MIND*

The uncertainties about climate sensitivity and climate damage are described by probability distribution functions. The information about climate sensitivity CS is modeled by a log-normal distribution by Wigley and Raper (2001): $\bar{\pi}(\mathrm{CS}) = \mathcal{LN}(0.973, 0.4748)$. The uncertainty about climate damage is taken to influence the amplitude a of the damage factor, but not the exponent b, which is taken as constant

$b = 2$. The distribution over a is derived from a normal distribution over the parameter a' in $DF(T)^* = 1/[1 + (T/a')^2]$, with $a' = \mathcal{N}(18, 5)$. This choice of the mean is near to the best guess case by Nordhaus (2008) ($a = 0.0028$ vs. our $a = 0.0030$). The uncertainty range is inspired by the distribution by Gerst *et al.* (2010), who chose $a = \mathcal{N}(0.0028, 0.0013)$, but due to the inverse distribution, higher damage values are favored by our distribution. For the numerical implementation we draw samples of size n from the distributions according to a scheme related to descriptive sampling (see Saliby, 1997). The uncertainty space is divided into n hypercubes. Each hypercube i carries a chosen probability weight w_i and is represented by the expected value of the parameters on this hypercube. For simultaneous uncertainty about both climate sensitivity and damage, each dimension is sampled with 10 equiprobable points.

Figure 2 shows the welfare changes, relative to the no-control case, for the different scenarios with and without uncertainty within the MIND model. First, the benefit from acting upon climate change is small relative to the net costs due to the existence of

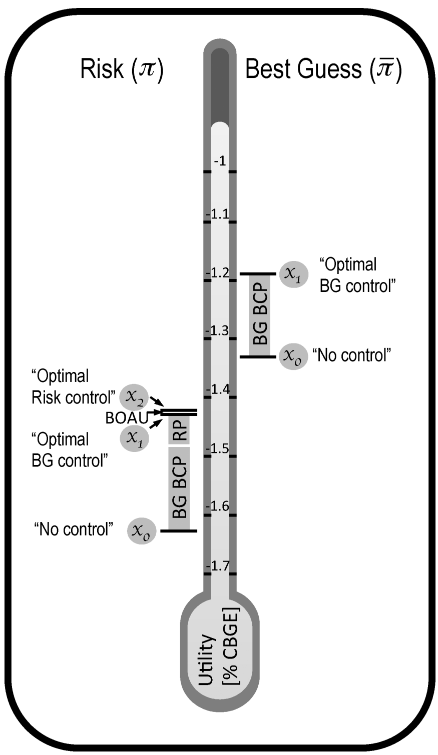

Figure 2. Welfare levels, measured as changes in CBGE relative to no-control, for the different scenarios (optimal climate policy in the best guess case x_1 and optimal climate policy under uncertainty x_2) with (π) and without ($\bar{\pi}$) uncertainty. Shown are the results for $\eta = 1.5$. The welfare differences represent the benefit of climate policy in the best guess case (BG BCP), the risk premium (RP) from reevaluating the optimal best guess policy under uncertainty, and the benefit of anticipating uncertainty (BOAU) from adjusting the optimal climate policy under uncertainty from x_1 to x_2.

climate change. In other words only a small part of the climate change induced welfare losses can be countered by mitigation policy. This observation stays the same in the uncertain setting, although the best guess climate policy leads to higher benefits against the no-control policy with uncertain damage. The welfare benefit from adapting the optimal policy is nearly invisible.

In a study close to this one, Pizer (1997) investigated the effect from explicitly including uncertainty into the DICE model by Nordhaus (1994). He not only considered uncertainty about the socio-economic and the climate system but also about the normative parameters of risk aversion and pure rate of time preference. He found that the uncertainty about the normative parameters by far dominates the uncertainties about the socio-economic system. We perform a sensitivity study of the uncertainty components towards the parameter of constant relative risk aversion. The necessary scenarios for optimal climate policies under best guess and uncertainty have been evaluated for 8 different values of η. The resulting changes in the partition of the benefit of optimal climate policy are depicted in Fig. 3. The changes in optimal decisions between the best guess and the uncertainty case are depicted in Fig. 4.

From Fig. 3, a clear ordering of the different components of the overall benefit of climate policy emerges: The main part of the overall benefit of climate action can be realized by simply taking the optimal best-guess policy. However, reevaluating this best-guess policy in an uncertain information setting significantly increases the benefit by adding a risk premium (RP). The changes in optimal decisions between the best guess and the uncertainty setting (see Fig. 4) are at least partly significant, e.g., a $>5\%$

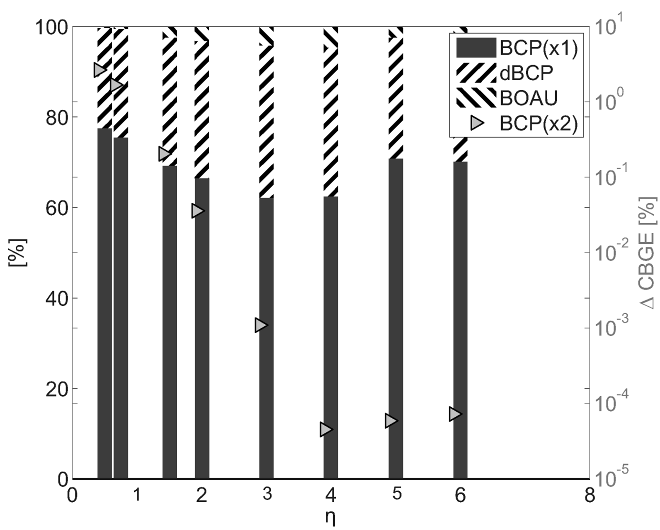

Figure 3. The three contributions (benefit of best guess policy in grey, reevaluation of best guess policy in left diagonal stripes (RP), benefit of adapting policy in right diagonal stripes) normalized to the overall benefit of climate policy under uncertainty, calculated for different values of constant relative risk aversion η (the three small values for $\eta \leq 2$ are $\eta = .5; .75; 1.5$). Also shown is the overall benefit of climate policy itself on the right axes.

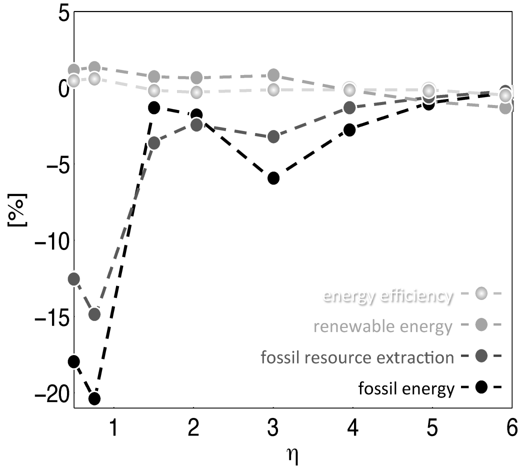

Figure 4. Relative changes in decision variables $\Delta\hat{x}$ (investments in R&D to increase energy efficiency, investments in extraction of fossil energy resources, investments in the capital stock of fossil energy carriers (infossil), and investments in the capital stock of renewable energy carriers, cumulated over the full time horizon (2010–2200), from the optimal best guess strategy $\hat{x}_1$ to the optimal strategy under explicit inclusion of uncertainty $\hat{x}_2$.

change in cumulative carbon emissions for the next two centuries. But the resulting welfare effect from this adjustments (BOAU) is insignificant for the whole range of η, thus the explicit incorporation of uncertainty into the optimization only plays a minor role. Summarizing the numbers indicates that within MIND uncertainty about the climate response to anthropogenic carbon emissions and about climate induced, quadratic damage is not important for the assessment of optimal climate change mitigation. An additional interesting feature of the sensitivity study with respect to relative risk aversion η is the rapidly decreasing overall benefit of climate policy for increasing η. For values of $\eta > 2$ the benefit from acting upon climate change gets lower than 0.01% of change in CBGE consumption. This can be explained with the dual role of the parameter η.

Within the expected utility framework employed in most studies of optimal global mitigation assessment the parameter η represents both, the DM's constant relative risk aversion and her aversion to fluctuations of consumption over time. With increasing risk aversion, the DM reacts with stricter policies to minimize the uncertainty in climate impacts. But with increasing aversion to consumption fluctuations over time, within an overall growing economy the DM's incentive to shift consumption from the future towards the present becomes stronger. Within MIND, obviously the second effect is stronger, as the mitigation effort decreases with η, and therefore the benefit from acting upon climate change also decreases. This finding is well known in the literature (see e.g., Kaufman, 2012; Ha-Duong and Treich, 2004). However the attempts to decouple both roles within numerical integrated assessment models have so far been unsuccessful in that they base on a decoupling of risk aversion and elasticity of intertemporal substitution that introduces a normatively unappealing preference for the timing of uncertainty resolution (Traeger, 2009). Potentially, a separation of both roles of η can be achieved within a normatively satisfying setting by Traeger (2009, and references therein). This is left for future studies.

In terms of the importance of explicitly including uncertainty, the other studies cited above have not investigated the separate effects of the risk premium for the optimal best guess policy and the marginal risk premium of adapting the optimal policy. Rather they investigated inhowfar prespecified policies are favored by the introduction of uncertainty. Thus, even if a decoupling of risk aversion and EIS generally leads to a higher risk premium, the question of the marginal risk premium for adjusting the optimal policy is still non-trivial.

3.3. *The marginal cost-benefit picture of MIND*

In this section we apply a marginal cost-benefit picture to the MIND model to understand the reasons for the small welfare effect from explicitly including uncertainty about the climate response and climate induced damage. Therefore, we interpret the welfare benefit from choosing the optimal policies $\hat{x}_1, \hat{x}_2$, instead of the no-control policy $\hat{x}_0$, as composition of mitigation benefits $B(x, \theta)$ and mitigation costs $C(x)$. We define mitigation costs $C(x)$ of policy x as the loss in welfare for choosing a suboptimal policy x instead of the optimal policy $\hat{x}_0$ in a world without climate damage (π_0): $C(x) \equiv [U(\hat{x}_0, \pi_0) - U(x, \pi_0)]$. The benefit of mitigation comes from a reduction in climate induced damages by adjusting the policy from the "no-control" action x_0 to a stricter mitigation regime x. The damages for the no-control case are given by $D(\hat{x}_0) \equiv U(\hat{x}_0, \pi) - U(\hat{x}_0, \pi_0)$, the welfare difference between the case with (π) and the case without (π_0) climate damages. The damages for the stricter mitigation policy x are given as $D(x) \equiv U(x, \pi) - U(x, \pi_0)$, respectively. Thus, for any climate policy x we define $B(x, \pi) \equiv \{[U(x, \pi) - U(x, \pi_0)] - [U(\hat{x}_0, \pi) - U(\hat{x}_0, \pi_0)]\}$ as the difference in the welfare impacts from the existence of climate induced damages between the policies x and $\hat{x}_0$. Simple calculus shows, that this choice actually delivers the desired composition for any policy x:

$$\begin{aligned} B(x, \pi) - C(x) &= \{[U(x, \pi) - U(x, \pi_0)] - [U(\hat{x}_0, \pi) - U(\hat{x}_0, \pi_0)]\} \\ &\quad - [U(\hat{x}_0, \pi_0) - U(x, \pi_0)] \\ &= U(x, \pi) - U(\hat{x}_0, \pi). \end{aligned}$$

Using this composition, the problem of finding the optimal climate policy $\hat{x}$ for a given information setting Eq. (1) can be rewritten as maximizing the difference between mitigation benefits and costs. This can be recast in an a-temporal cost-benefit picture by identifying the intersection of the marginal benefits $(dB(x, \pi)/dx)$ and marginal costs $(dC(x)/dx)$. For general multidimensional decision variables x these marginals are the total derivatives along all dimensions. To be able to inspect the cost-benefit picture visually we additionally need to project the multi-dimensional decision variable x on a single-dimensional quantity. Thereby, we lose the exact equivalence

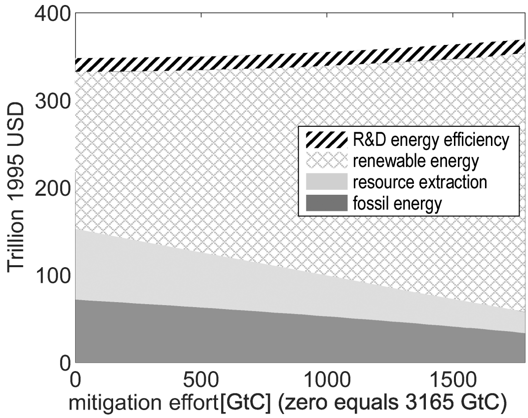

Figure 5. Aggregated investments (NPV, discounted with 5%) into the energy system depending on the required mitigation effort (in GtC cumulative reduction relative to the no-control in which 3165 GtC are emitted over the period 2010–2200).

between the welfare picture and the cost-benefit picture. The goal is to choose a projection $x \rightarrow \tilde{x}$ that approximates the welfare effect of uncertainty with high accuracy and allows an interpretation of the small amplitude. We achieve the one-dimensional projection by introducing a constraint on cumulative emissions in a setting without climate damage. For a constraint above 3165 GtC the no-control policy emerges. With decreasing levels of admissible cumulative emissions, the DM reacts by adjusting the investments into the different energy technologies (see Fig. 5). With increasing stringency of the constraint on cumulative emissions the investments into R&D in energy efficiency increase, as well as the investments into carbon free renewable energy. Contrary to this, the investments in carbon intensive fossil energy carriers and the corresponding resource extraction sector decrease.

Introducing a dense sampling in the cumulative emissions constraint and evaluating the resulting policies in settings with and without uncertainty allows to construct the cost-benefit picture for the MIND model (see Fig. 6). The marginal benefits for the best-guess case are derived by fixing all uncertain parameters to their expected value while the expected marginal benefits are derived by applying the cost-benefit decomposition to the expected utility. The fluctuations in the gray curves that represent the raw data from the model can be explained by the limited temporal resolution of the model (5 years). When optimizing under a binding constraint with increasing stringency (such as the constraint on cumulative emissions), the timing of the mitigation effort to stay below the constraint can only be adjusted within this limited temporal resolution. This leads to small jumps in the overall welfare and thus also in marginal welfare and in marginal benefits and costs. The bold black lines are polynomial fits to the raw data.

The optimal mitigation effort for the best-guess and the uncertainty setting can be obtained as intersections between the marginal costs and the (expected) marginal

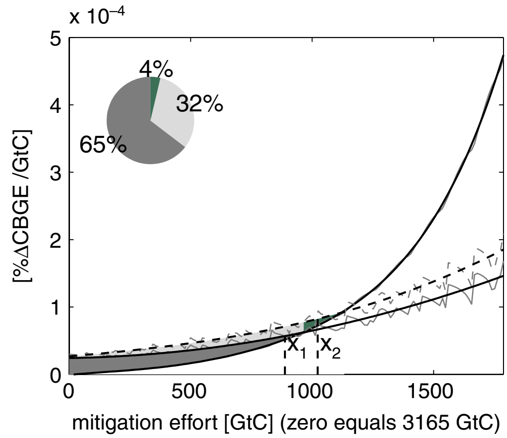

Figure 6. Marginal costs (lower black line) and (expected) marginal benefits (upper black and black dashed line) of mitigation within the MIND model. The gray curves are the raw data from MIND, the black curves are polynomial fits. Mitigation effort is parametrized by a decreasing constraint on cumulative emissions. The optimal policies from the welfare optimization are shown as vertical dashed lines.

benefits of mitigation. The different contributions to the overall benefit of acting upon climate change can be visualized as areas between the benefit and cost curves. The pie chart in the upper left corner shows their relative contributions to the overall benefit of climate policy. A comparison between the optimal values of cumulative emissions derived from the marginal picture and those derived from the welfare optimization shows the "error" of the approximation. The optimal level of mitigation in cumulative emissions is represented within an 4% error, while the welfare effects of uncertainty are overestimated by up to 5%. The applicability of our diagnostic to other models rests on finding such a valid projection of the multi-dimensional decision space onto a scalar decision variable, that is able to produce policies resulting from the marginal cost-benefit assessment that are close to the policies derived from the welfare optimization.

Nevertheless, the cost-benefit picture allows to identify both, reasons for the negligible uncertainty effect within the MIND model and general conditions for a significant uncertainty effect. First, within MIND the overall value of acting upon climate change is constrained due to the convex increasing functional form of both (expected) marginal benefits and marginal costs combined with a similar curvature and the small initial (for zero mitigation) difference between both curves. The combination of these conditions leads to a very small area between both curves and thus, a small overall benefit of climate policy. The convex increasing (expected) marginal benefits are somewhat counter intuitive as one would assume climate damage to be convex increasing in temperature and temperature more or less linearly connected with mitigation effort and thus would expect decreasing marginal benefits. The reason for the counter-intuitive finding from the MIND model is discussed further in the following pages.

Second, within MIND the marginal risk premium, that is the difference between expected marginal benefits under uncertainty and marginal benefits for the expected parameter values, is small for zero mitigation, only increases linearly in the mitigation effort, and with a small slope. Together with the strongly convex increasing marginal costs, this leads to a relatively small difference between the two optimal policies $\hat{x}_1$ and $\hat{x}_2$. Thus the area representing the benefit of anticipating uncertainty (BOAU) is small, both, in absolute terms and compared to the overall benefit of acting upon climate change.

In general, there are a number of options to increase the BOAU in absolute terms: One would need to either increase the difference between $\hat{x}_1$ and $\hat{x}_2$ or the marginal risk premium for all x between $\hat{x}_1$ and $\hat{x}_2$. This could be achieved by a general upward shift of the marginal risk premium (adding a constant), by increasing the slope of the MRP, or by increasing the convexity of the MRP in the mitigation effort. Similarly, a decrease in the convexity of the marginal mitigation costs would increase the difference between $\hat{x}_1$ and $\hat{x}_2$ and thus the BOAU. Even a decreasing (marginal) benefit of mitigation could lead to a larger absolute BOAU, if combined with a larger MRP.

The more important question however is for the conditions of a significant BOAU in relative terms with respect to the overall benefit of climate policy. A few general statements are evident from observing Fig. 6: Everything else being equal, a less convex marginal mitigation cost curve would increase the BOAU in absolute terms and at the same time decrease the benefit of the optimal climate policy in the best guess case and thus would increase the BOAU in relative terms. For constant $\hat{x}_1$ and $\hat{x}_2$ a decrease in the slope (i.e., a negative slope) of the (expected) marginal benefit of mitigation would increase the benefits of the optimal best guess policy and the re-evaluation effect and thus decrease the BOAU in relative terms. An increased slope and convexity of the marginal risk premium would unambigously increase the relative importance of the BOAU. A constant upward shift of the MRP however could lead to both, an increase or a decrease of the relative importance of the BOAU, depending on the relative position of $\hat{x}_1$ and $\hat{x}_2$ with respect to the zero mitigation point.

3.4. *Functional dependencies within MIND*

To find an explanation for both results, the slope and curvature of marginal benefits and hence small BOAU, we apply the marginal representation to the single steps in the climate cause-effect chain. The absolute and marginal functional form of the individual elements of the chain are shown in Fig. 7. This allows us to investigate in detail how the slope and curvature are determined in the integrated assessment model MIND.

The (cumulative) emissions lead to a rising concentration of greenhouse gases in the atmosphere and increasing radiative forcing. The maximum forcing reached for different levels of mitigation effort is shown in Fig. 7(a). The maximum total forcing is concave increasing in cumulative emissions and concave decreasing in mitigation effort respectively. This can be explained by the saturation effect represented by the

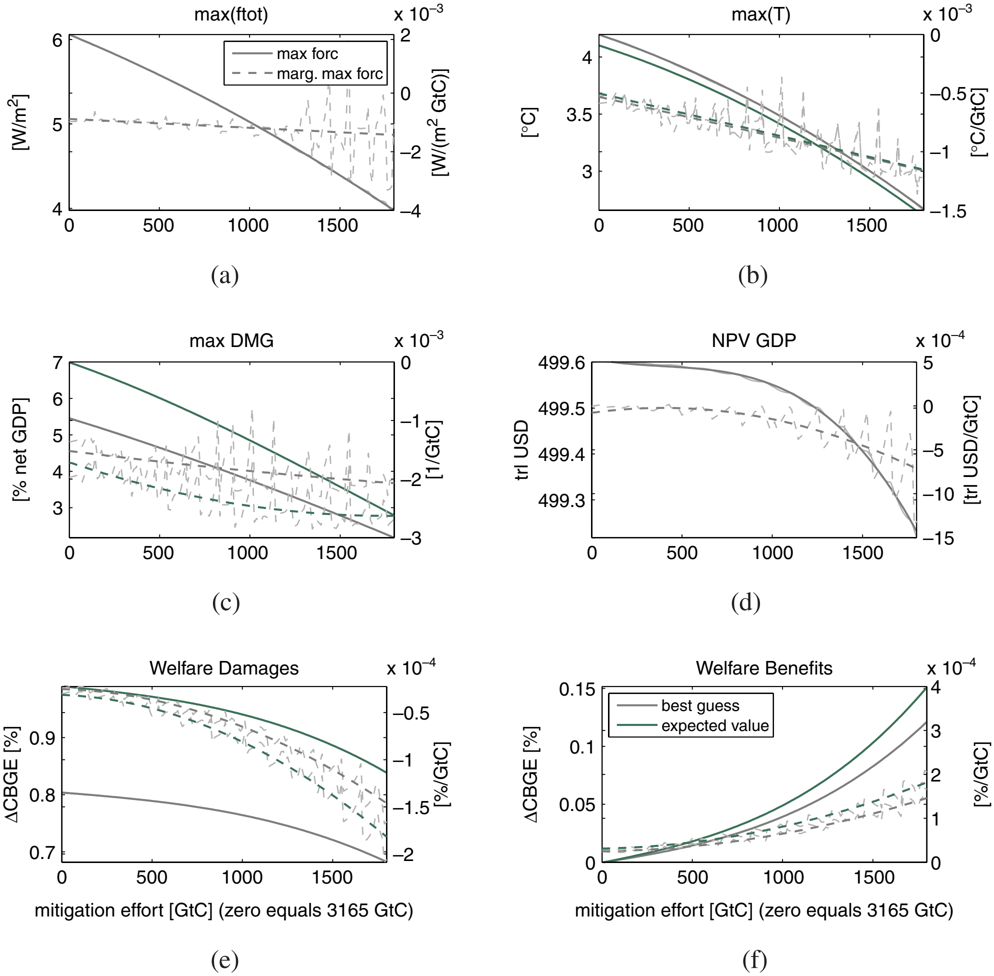

Figure 7. Functional dependencies between mitigation effort (measured in terms of cumulative emissions reductions from the BAU emissions of 3165 GtC in the period 2010–2200) and individual components in the cause-effect chain of climate change: (a) maximum radiative forcing, (b) maximum temperature change, (c) damage in % of net GDP for the maximum temperature change, (d) net present value (NPV) of gross output including mitigation costs, but excluding climate damage, (e) welfare equivalent damage measured in %ΔCBGE, and (f) welfare benefits measured in %ΔCBGE. Shown are the functions (continuous lines) and the marginal functions (dashed lines). For those quantities that depend on the uncertain SOW the best guess value is shown in dark grey and the expected value of the uncertain setting is shown in darker grey. The original model data are shown in lighter grey, where as the functional dependencies are polynomial fits of the data.

logarithmic relation from concentration to forcing. With increasing atmospheric concentration of carbon dioxide, the frequency band in which CO_2 absorbs the outgoing radiation saturates, thus a further increase in concentration leads to less and less additional radiative forcing. The same concave behavior, increasing in cumulative emissions and decreasing in mitigation effort, occurs for the maximum temperature increase max (T), shown in Fig. 7(b). The global mean temperature reacts to changes in radiative forcing. The overall temperature response is determined by the amplitude (climate sensitivity) and time scale (ocean diffusivity) of a simple impulse response model. The climate damage that is incurred by the maximum temperature change, measured in % of net GDP, is shown in Fig. 7(c). Figure 7(d) shows the net present value of gross output excluding climate damage, aggregated over time by an endogenous discount rate $\rho_t \equiv \delta + \eta \cdot g_t$, where δ is the pure rate of time preference, η is the rate of constant relative risk aversion and g_t is the endogenously determined growth rate of consumption. The gross economic output is concave-decreasing in the mitigation effort, leading to convex increasing mitigation costs in GDP terms, which are derived by subtracting the gross output curve from the gross output of the no-control case $x = 0$.

Multiplying the damage factor, $\mathrm{DF} = 1/(1 + D)$, where D are the net GDP damage from Fig. 7(c), with the gross GDP in each time step gives the time series of net GDP that constrains the investment decisions and consumption level via a budget equation. Thus both the costs from mitigation (as seen in the gross GDP) and the climate damage lower the consumption level and thus the welfare. The welfare equivalent damage for the different mitigation scenarios, shown in Fig. 7(e), is derived by evaluating the difference in CBGE between a case with the damage factor DF as above and a case without damage (but with mitigation costs), where $\mathrm{DF} = 1$. Formally, the welfare damage is given as $\Delta\mathrm{CBGE}(V(x, \pi_0), V(x, \pi/\bar{\pi}))$ or in loose notation as $U(x, \pi_0) - U(x, \pi/\bar{\pi})$. Normalizing the welfare damage to the no-control case delivers welfare benefits from mitigation, shown in Fig. 7(f). Formally, this normalization is done by subtracting the welfare damage for the no-control case, leaving us with the definition of benefits from Sec. 3.3; $U(x_0, \pi_0) - U(x_0, \pi) - (U(x, \pi_0) - U(x, \pi)) = B$.

The two most interesting features in the cost-benefit picture of MIND are the positive slope and the convex curvature of the marginal benefits of mitigation. Concerning the slope of the marginal benefits in welfare, the explanation can already be found in Figs. 7(b) and 7(c). The marginals of maximum (Fig. 7(c)) and welfare equivalent damage (Fig. 7(e)) are decreasing in the mitigation effort implying increasing marginal benefits. As can be seen from a comparison of Figs. 7(c) and 7(e), this behavior is not a result of the welfare evaluation of climate damage (although it is strengthened by it), but already present in the marginal of the maximum damage. Since the maximum damage is convex-increasing with rising temperature, their marginal is increasing with temperature as well. The fact that their marginal is decreasing when plotted against increasing mitigation effort instead of temperature, due to concave instead of convex decreasing maximum damage, points to the fact that the concavity found in the temperature response to mitigation dominates the convexity of damage in rising temperature. Thus, we find that the saturation of the emissions to temperature change relationship over-compensates the non-linearity in the climate damage function, leading to increasing instead of decreasing marginal benefits of mitigation, and thus limiting the overall benefit of the best-guess climate policy.

The convexity of marginal welfare benefits in mitigation effort however does not originate from the combination of maximum temperature with the damage function, but emerges from the welfare valuation of climate damage. This can be seen by comparing the convex decreasing marginal of maximum damage (Fig. 7(c)) to the concave decreasing marginal of welfare equivalent damage (Fig. 7(e)). Hence, the influence of the welfare function, i.e., of the normative parameters of constant relative risk aversion η and pure rate of time preference ρ determines the curvature of the marginal benefits. Comparing the marginal benefits for the best-guess case and the case of uncertainty, it can be seen that the convexity increases when accounting for uncertainty, implying a convex increasing MRP. Thus, the additional marginal welfare benefit of reducing a unit of emissions under uncertainty grows with increasing mitigation effort. This works against a large contribution of re-evaluating the best guess climate policy under uncertainty (RP), and favors a larger relative contribution of adjusting the mitigation policy under uncertainty (BOAU). However, due to the strongly increasing marginal mitigation costs, the welfare gains from adjusting the mitigation policy remain small.

4. Changes in the Model Structure

Which assumptions about the climate cause-effect chain would lead to a significant welfare gain from adapting the optimal policy to uncertainty? In this section, we investigate several changes in the model structure and their influence on the cost-benefit picture and the BOAU.

4.1. *Constant relative risk aversion* η

We have shown in Sec. 3.4 that the curvature of the welfare function, represented by the parameter of constant relative risk aversion η, strongly influences the curvature of the marginal benefits of mitigation. We have also shown that the curvature of the marginal benefits strongly influence the overall benefit of climate policy. We use these dependencies and investigate the relative importance of adjusting the optimal policy to uncertainty depending upon the parameter of constant relative risk aversion. The changing cost-benefit pictures of MIND are shown in Fig. 8 for values of η between 0.75 and 3.

The effects of the curvature of the welfare function are manifold: The first, and most important one, is a scaling effect. As already shown in Fig. 3, the overall net benefit of climate policy strongly decreases with increasing η. This effect can be explained by the dual role of η. It does not only represent risk aversion, but also the DM's aversion towards inter temporal fluctuations in consumption. If this aversion is high, the DM prefers a smooth, constant consumption stream over a fluctuating, increasing one. In a growing economy with consumption growth in the future the decision maker prefers to delay mitigation as it would require to divert consumption into early investments in carbon free energy technologies. This effect can already be seen in the baseline

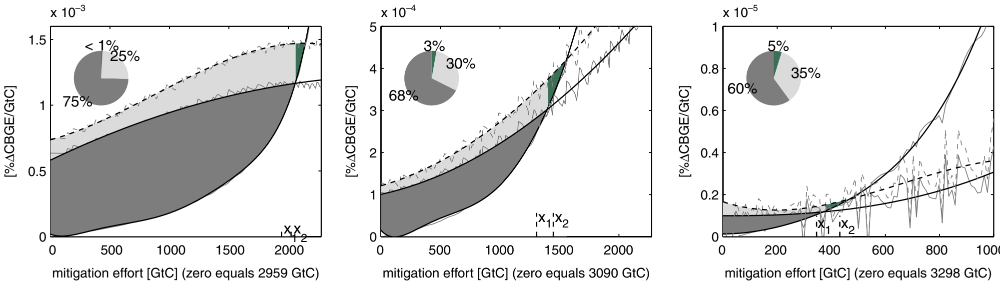

Figure 8. Sensitivity of the marginal cost–benefit picture of MIND with respect to changes in the parameter of constant relative risk aversion η. Shown are the pictures for $\eta = 0.75$(left), $\eta = 1.5$(middle), $\eta = 3$(right). The legend is equivalent to Fig. 6.

cumulative emissions (the number given as label below the figures). Hence the net benefits from mitigation, i.e., reduced climate damage minus mitigation costs, are lower for a high η as mitigation reduces early consumption. The second effect concerns the curvature of the marginal benefits and thus the curvature of the marginal risk premium. This effect is directly evident: The exponent η of the welfare function obviously directly impacts on the curvature of the marginal welfare depending on consumption.

In combination both effects result in increasing absolute overall benefits of climate policy and increasing absolute welfare gains from adjusting climate policy to uncertainty with decreasing η. However, the relative contribution of the BOAU to the benefit of climate policy decreases with decreasing η.

Thereby, of course, we acknowledge the limitations of the expected utility framework that links both roles of η together, as well as the fact that empirical estimates for relative risk aversion are more in line with values of 3–10 (Siman *et al.*, 2009). Hence, what the changing marginal cost-benefit picture of MIND (with changing η) should demonstrate is not the importance of explicitly including uncertainty within a realistic setup but rather it should demonstrate the point that the question of the importance of explicitly including uncertainty will still be nontrivial even with a more appropriate representation of risk aversion.

4.2. *Exponential damage*

Unlike Weitzman (2010), who focused on the potential fat tails of the distribution on climate sensitivity and climate damage we are searching for a setting in which a strong impact of uncertainty on optimal mitigation efforts also occurs for thin tailed distributions. As stated before, a stronger increase, and convexity, in the marginal risk premium in welfare terms for rising mitigation effort would lead to a stronger (relative) BOAU.

First, we replace the standard quadratic formulation of the damage function by an exponential formulation:

$$\mathrm{DF}_e = \frac{1}{1 + k \cdot \exp\left(\frac{T}{l}\right) - k}. \tag{9}$$

We choose the parameters k and l such that the exponential damage in net GDP, $k \cdot \exp(T/l) - k$ equal the standard formulation at $T = 3^{\circ}$ for the best guess case. We assume a normally distributed l with $l = \mathcal{N}(2.2571, 0.61)$. Together with $k = 0.01$, this choice leads to a best guess marginal damage function which is nearly identical to the standard best guess marginal damage function used in the previous section. However, the expected marginal damage function is far more convex in temperature than in the quadratic case. Thus the marginal risk premium in net GDP damage increases more strongly. The resulting difference in the marginal of the maximum damage is shown in Figs. 9(a) and 9(b) together with the identical maximum

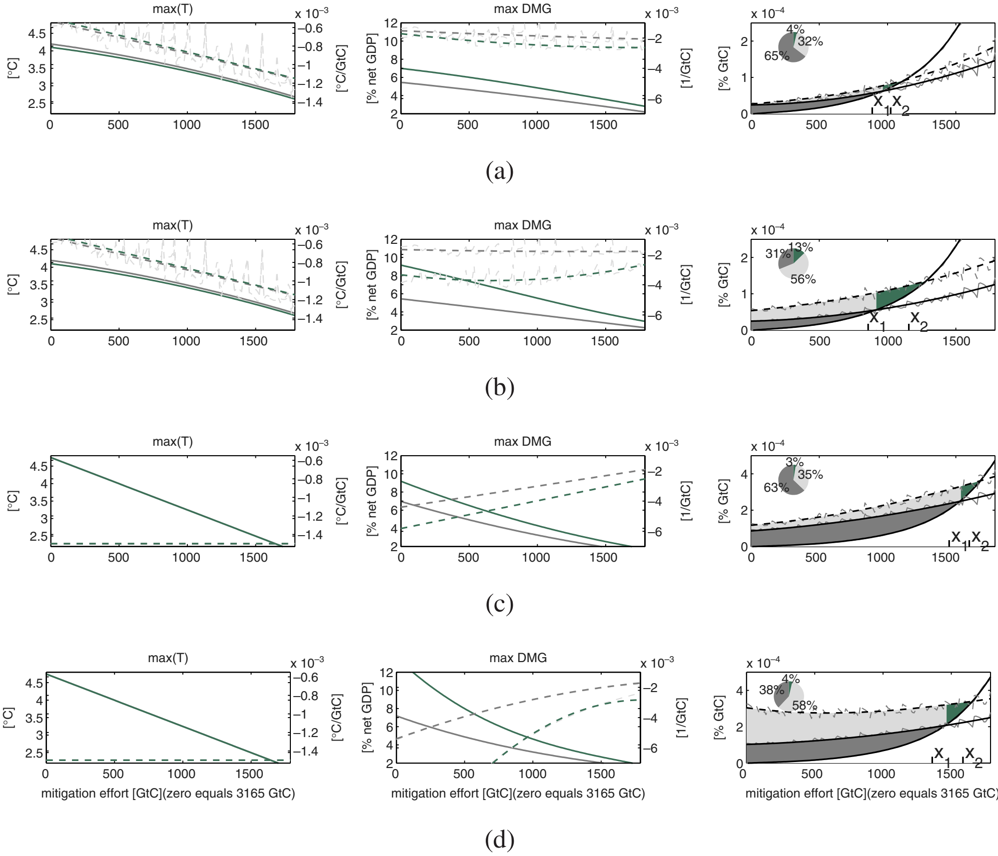

Figure 9. Maximum temperature change, maximum climate damage in net GDP, and marginal cost benefit picture for four different structural model settings: (a) standard climate module and quadratic damage function, (b) standard climate module and exponential damage function, (c) linear climate carbon response and quadratic damage function, and (d) linear climate carbon response and exponential damage function. The functional relations are shown in grey for the best guess case and in darker grey for the expected value of the uncertainty case. The dashed lines represent the marginal functions. The fluctuating lines in lighter grey are the original model data. The smooth lines are polynomial fits to the data. The legend for the cost benefit pictures is analogous to Fig. 6.

temperature functions and the resulting cost benefit pictures. The change towards exponential damage shows several interesting effects: First, as we have chosen identical best-guess marginal damage, the optimal policy in the best-guess case also does not change, but the higher marginal risk premium (shifted upwards) leads to an increased $\hat{x}_2$. The welfare benefit from adapting the policy to uncertainty increases significantly both in absolute and relative terms and now contributes 13% to the overall benefit of climate policy (instead of 4% in the quadratic case). The increase in the relative contribution of the BOAU is dampened by the fact that the effect of reevaluating the best-guess policy under uncertainty (RP) is also strongly increasing. This is due to the fact that the expected marginal benefits do not only increase more strongly than before but are also shifted upwards over the whole domain. Second, the change towards exponential damage leads to at least partly increasing expected marginal damage in net GDP. However this shift in the slope of the marginal damage is not strong enough to be reflected in the expected marginal benefits, it "gets lost" through the convolution with the welfare function. Finally, compared to the case of quadratic damage, the overall value of climate policy more than doubles for the assumption of exponential damage, with more than two thirds of the benefit due to taking into account uncertainty.

4.3. *Linear carbon climate response*

Finally, we replace the climate module by a linear relationship between cumulative carbon emissions and increase in global mean temperature that has been found by Matthews *et al.* (2009) within an ensemble of state of the art climate models. By introducing the so called carbon climate response (CCR) parameter, the relationship between global mean temperature change (relative to pre industrial) ΔT and cumulative carbon emissions reads:

$$\Delta T(t) = \mathrm{CCR} \cdot \sum_{t_0}^{t'} e(t'), \tag{10}$$

where e represents globally aggregated carbon emissions. Within the model ensemble, Matthews *et al.* (2009) found a carbon climate response of $\mathrm{CCR} = 1.5[1.0$–$2.1]\,^\circ\mathrm{C}/\mathrm{TtC}$. The values in square brackets mark the 5 and 95 quantile. We choose a log-normal distribution for the CCR with $\mathrm{CCR} = \mathcal{LN}(\log(1.461), 0.23)$, which gives the best fit to the quantiles and 1.5°C as expected value. The resulting maximum temperature, maximum net GDP damage and the cost benefit pictures are shown in Figs. 9(c) and 9(d). In Fig. 9(c) the linear climate carbon response is combined with quadratic damage and in Fig. 9(d) with exponential damage from Sec. 9. Considering the maximum temperature response, the difference between the best guess case and the expected case under uncertainty nearly vanishes. This is clear, as the uncertain parameter CCR now enters linearly into the function, thus the expectation operator only acts on the parameter itself. Thus, the uncertainty in the climate system is now irrelevant for the mitigation problem. But even more interesting is the change in the best-guess maximum temperature function itself. It declines more strongly in the mitigation effort than before. This leads to a stronger difference between the best guess and expected damage. The changed curvature of the climate response leads to convex decreasing net GDP damage. However, the increasing slope of the marginals is changed back to decreasing marginal welfare equivalent damage further downstream by the welfare function. Hence the marginals in the welfare benefit are still increasing, but less convex than before. Compared to the standard climate module, the overall benefit of climate policy increases strongly, but the individual contributions of the three components remain largely unchanged. In particular, the relative welfare contribution from adapting the optimal policy to uncertainty is still negligible.

Combining the linear climate carbon response with exponential damage amplifies the distinct features of the two cases. The convexity in expected net GDP damage gets strong enough to "survive" the convolution with the welfare function leading to initially decreasing expected marginal benefits in welfare terms. Thus, inspite of a high marginal risk premium, this further increases the benefit from reevaluating the best-guess policy. The BOAU stays small in relative terms.

Summarizing the results from this section, the functional formulation of the building blocks of the climate cause-effect chain (temperature response, climate induced damage and the aggregated welfare function) and especially their marginals determine the relative strength of the effect of including uncertainty. Thereby the non-linearities in the temperature response and the damage function partly compensate each other. Everything else equal, the importance of adjusting policies to uncertainty becomes the more important (relative to the overall benefit of climate policy) the more convex the damage function, the lower the risk aversion parameter and the lower the concavity of the maximum temperature in cumulative emissions.

5. Conclusion

This study adds two important points to the literature that investigates the importance of explicitly introducing uncertainty into integrated assessment models of climate change: First, based on the integrated assessment model MIND, a terminology and an analysis diagnostic are developed which allow to answer the question of importance of adjusting optimal climate policy to uncertainty. Even independent of the question of the importance of explicitly including uncertainty the proposed analysis diagnostic is of value to understand the interaction of the different functional representations of the steps of the cause effect chain within complex integrated assessment models. Thus, it takes us one step away from the "black-box mode" in which the integrated assessment model is only tested as a whole. Second, within the integrated assessment model MIND the application of the diagnostic tool can explain the negligible welfare effect (relative to the overall benefit of climate policy) from adjusting optimal mitigation efforts not from the absense of structural elements, like fat-tailed uncertainty, climate catastrophies or a more realistic preference representation, but from the functional structure of the cause-effect chain of climate change within the model.

More specifically, we applied a decomposition of the overall benefit of acting upon climate change into its single components to measure the relative importance of uncertainty within the integrated assessment model MIND. Uncertainty influences both the optimal mitigation policy and the expected utility of different policies. Including uncertainty explicitly is important, if it leads to a significant change in the optimal policy that in turn leads to a significant change in the benefit gained from acting upon climate change. Uncertainty might also be considered relevant, but would not have to be included explicitly into the optimization framework, if it significantly changed the assessment of the benefit of climate policy compared to the best guess case (risk premium), even though the optimal climate policy did not change significantly. Within the MIND model the risk premium for the optimal best guess policy is dominating the welfare gain from adjusting the policy under uncertainty. Overall, the welfare effect of accounting for uncertainty (relative to the overall benefit of climate policy) is rather small, which further corroborates the findings in the literature.

To understand the origin of these findings, we projected the complex MIND model to an a-temporal marginal cost-benefit picture and resolved the functional relationship between the single steps of the climate cause-effect chain. We then located the origin of the negligible welfare gain from adapting the optimal policy to uncertainty. This benefit of anticipating uncertainty (BOAU) is only significant if uncertainty leads to a strongly convex increasing marginal risk premium with increasing mitigation effort. In the standard model setting with a quadratic damage function and a zero-dimensional climate-carbon response box model, this behavior was constrained by the saturation of the emissions to temperature change relationship compensating for the non-linearity in the climate damage function and by the consumption smoothing property of the welfare function. Thus, for seeing a significant influence from including uncertainty one has to consider alternative model settings that induce a strongly convex increasing expected marginal benefit from mitigation (and thus a convex increasing MRP). Two such changes in the model setup, an exponential climate damage function and a linear climate carbon response have been implemented. We showed that those changes to the model structure indeed can lead to a significant uncertainty effect (and a convex increasing MRP). The other feature that constrains the importance of including uncertainty is the strongly increasing marginal mitigation cost curve in the model. Thus, a change in the convexity of marginal mitigation costs, especially reducing the strong increase for higher levels of mitigation would also lead to more significant uncertainty effects. This is especially important when including changes to the model structure, like introducing strongly non-linear climate damages that could represent tipping-points in the climate system. Whether or not this leads to a higher relative importance of explicitly including uncertainty depends not only on the level or risk aversion and the distribution of uncertainty about the tipping points but also on the curvature of mitigation costs. If mitigation costs are also increasing non-linearly around the same level of mitigation that would avoid triggering the tipping-point, the combined uncertainty effect might still be insignificant. This is of special interest as it emphasizes the combined importance of the modeling of mitigation options and the impact and damage formulation for the overall importance of uncertainty for the integrated assessment of climate change.

These results come with the usual caveats. The employed integrated assessment model MIND, although more complex than quasi-analytical cost-benefit models and the commonly used DICE model, still includes a strongly simplified representation of the cause-effect chain of climate change. The representation of uncertainty had to be constrained to a few sample points and we only investigated the effect from a single information setup. Thus this study should not be seen as an attempt to find a conclusive answer to the question whether accounting for uncertainty is important for the assessment of climate policy. Rather, we present an approach to decompose and trace the uncertainty effect in complex integrated assessment models that complements former studies of structural changes that can induce strong uncertainty effects. The general applicability to other complex numerical integrated assessment models rests on finding a valid projection of the multi-dimensional decision space onto a scalar decision variable such that the policies derived from a marginal cost-benefit analysis are close to those policies derived from a welfare optimization. This remains to be shown. We believe our approach will prove useful to these other examples as well as it emphasizes the necessity of investigating the complete (expected) marginal benefit curves and optimal policies instead of just comparing a small number of sample mitigation policies. Thereby it will improve our understanding about the effect of structural model assumptions on the significance of the uncertainty effect.

Appendix

A comparing welfare across different scenarios

As (expected) utility is only defined up to an affine transformation, we use differences in the certainty and balanced growth equivalents (CBGE), as presented by Anthoff and Tol (2009), to compare different scenarios. The certainty equivalent of an uncertain consumption outcome is an amount of consumption the DM would demand instead of a distribution of outcomes to get the same expected utility. The same principle works for the balanced growth equivalent: Here the consumption path, that possibly varies over time, is replaced by a path consisting of an initial consumption level that growth over time with a constant growth rate α and gives the same utility. If one is only interested in relative changes in the CBGE between different scenarios, the measure is independent of the growth rate α. Thus, the relative change in CBGE, denoted by ΔCBGE, can be interpreted as fraction of consumption the DM would be willing to pay, now and forever, to switch from a scenario with lower CBGE to the other scenario. Formally the ΔCBGE for isoelastic utility reads:

$$\Delta\mathrm{CBGE}[EU_1, EU_2] \equiv \begin{cases} \left(\dfrac{EU_1}{EU_2}\right)^{1-\eta} - 1 & \text{for } \eta \neq 1 \\ \exp\left(\dfrac{EU_1 - EU_2}{\sum_{t_0}^{T} P_t (1+\rho)^t}\right) - 1 & \text{for } \eta = 1 \end{cases}. \tag{A.1}$$

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