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Abstract

This paper investigates the strategic interactions between carbon taxation by a resource-consumers' coalition and (wellhead) energy pricing by a producers' cartel under possible innovation in a cheap carbon-free technology through a dynamic game. The arrival time of innovation is uncertain, but can be affected by the amount spent on R&D. The results show that the expectation of possible innovation decreases both the initial carbon tax and producer price, resulting in higher initial resource extraction or carbon emissions. Even though this 'green paradox' effect will appear in the cooperative case (no strategic interactions) as well, the presence of strategic interactions between resource producers and consumers can somewhat restrain such an effect. The optimal R&D to stimulate innovation is an increasing function of the initial CO2 concentration for both the resource consumers and a global planner. However, the resource consumers can over-invest in R&D (compared with the global efficient investment).

Keywords: Carbon taxation; Innovation; Uncertainty; Dynamic game
JEL Classification: C73, Q23, H21, Q54

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1 Introduction

The potential climate change caused by global warming has been considered as one of the most important environmental issues in this century. Many scientists believe that the observed temperature increase on the earth is a result of the accumulation of greenhouse gases (GHGs) in the atmosphere and it can bring severe damages to the human society and ecosystems. Mitigating climate change would require an appropriate design of climate policies, for instance, carbon taxes, to take into account the externalities caused by GHGs emissions, which mainly come from fossil fuel consumption.

Though carbon taxes can be adopted to mitigate global warming, their usage is controversial especially when the fuel prices are already high. Some people argue that carbon taxes should be reduced in response to increased fuel prices, for the benefit of consumers, while others believe that they should be increased further, to effectively abate carbon emissions. The strategic interactions between the (cartelized) resource producers and consumers make the arguments about carbon taxes more complicated. More specifically, an energy producer such as OPEC can react strategically and preempt carbon taxes by raising the producer price (Wirl, 1995). Meanwhile, a coalition of resource-importing countries such as the International Energy Agency (IEA) could coordinate their carbon taxation and thereby affect the pricing strategy of energy producers. Therefore, it is of great significance to investigate climate policy issues in the presence of this two-side strategic interaction, where the consumer side that is coordinating taxation understands the effect of carbon taxes on energy prices, and the producer side that is coordinating sales understands the effect of sales on taxation (Liski and Tahvonen, 2004).

In addition to the strategic interaction issues, the possible innovation of low-carbon or carbon-free technologies can also have important implications for climate policy design and may have an effect on the strategic interactions between carbon taxation and energy pricing. Imagine now that there is a possibility that a carbon-free technology (which is a perfect substitute for fossil fuels) can be invented or discovered at some time in the future and can be supplied at a lower cost (price) than that of fossil fuels. Since the new technology will have an effect on fossil energy consumption and CO_2 emissions, both the energy producers and the consumers need to take this into account. Then a natural question is, how would the producers and consumers change their strategies of energy pricing and carbon taxation with the expectation of possible innovation? How would the effect of possible innovation differ with/without the strategic interactions between the producers' energy pricing and and the consumers' carbon taxation? Moreover, if the arrival time of innovation can be affected by the consumers' strategic R&D effort, how would the consumers make their R&D decisions? How does the optimal R&D investment by the consumer side compare with the global efficient level? To investigate these questions, this paper integrates the possible innovation of a carbon-free technology into the strategic interactions between the energy seller side and the buyer side on energy pricing and carbon taxation within a dynamic game framework to study the role of possible innovation and R&D investment in this strategic interaction context.

While the role that technological innovation (and its uncertainty) plays in natural resource extraction or climate policy design has been investigated by numerous studies, e.g., Dasgupta and Stiglitz (1981), Harris and Vickers (1995), Golombek et al. (2010), Fischer and Sterner (2012), and Henriet (2012), the strategic interactions between climate policy design and resource extraction were generally not addressed in these studies. On the hand, even though the strategic interactions between (fossil fuel) producers' energy pricing strategies and consumers' carbon taxation have been extensively examined in the literature (for instance, Wirl, 1994, 1995; Wirl and Dockner, 1995; Tahvonen, 1994, 1996, 1997; Rubio and Escriche, 2001; Liski and Tahvonen, 2004; Wei et al., 2012), none of the previous studies (to the best knowledge of the author) has incorporated the possible innovation of carbon-free technologies, the uncertain arrival time of innovation, and the endogenous R&D investment, into the investigation of the strategic interactions on carbon taxation and energy pricing. This paper fills these gaps in the literature.

Another concept that is related to this paper is the so-called 'green paradox', which stems from Sinn (2008) and describes the situations in which some climate policies designed to abate carbon emissions might actually increase carbon emissions, at least in the short run (Hoel, 2012). For instance, a rapidly increasing carbon tax (Sinn, 2008), or the anticipation of a cheap and clean backstop technology (Henriet, 2012), can be the possible causes of a green paradox. In line with the 'green paradox' argument, this study finds that the expectation of possible innovation in a cheap carbon-free technology decreases both the initial carbon tax and initial producer price, which implies lower initial consumer prices and thus higher initial resource extractions or carbon emissions. Even though this effect can be found in the case without strategic interactions as well, the decrease in initial consumer price, and thus the increase in initial carbon emissions can be less dramatic in the presence of the strategic interactions of carbon taxation and energy pricing between the energy producer side and the consumer side, given that the environmental damage of cumulative emissions is sufficiently high. This result indicates that the 'green paradox' effect of possible innovation can be somewhat restrained by the strategic interactions between resource producers and consumers. Moreover, if the consumer side can affect the arrival time of innovation through R&D, it might exert an R&D effort that is higher than the global efficient level, provided that the environmental damage of cumulative emissions is sufficiently high.

The rest of this paper is organized as follows. Section 2 describes the dynamic game and derives the non-cooperative and cooperative strategies, respectively. The effect of possible innovation on players' strategies is analyzed in Section 3. In Section 4, the hazard rate of innovation is endogenized and optimal R&D for innovation is investigated. Concluding remarks and their policy implications are summarized in the final section.

2 The dynamic game

2.1 Model setup

As in Wirl (1995), Tahvonen (1994, 1996, 1997), Rubio and Escriche (2001) and Liski and Tahvonen (2004), there are two players in the dynamic game of strategic interactions: a consumers' coalition (such as an empowered International Energy Agency), which maximizes the net present value of consumers' welfare by choosing a carbon tax $\tau(t)$; and an energy producers' cartel (such as OPEC), which maximizes the net present value of profits by setting the wellhead energy price (i.e., producer price) $p(t)$. Consequently, the consumer price at time t would be $\pi(t) = p(t) + \tau(t)$, which will determine the (non-negative) consumption for fossil energy (measured in emissions) $D(t) = a - b\pi(t)$, where $a > 0$ and $b > 0$ are constants.

The concentration of CO_2 in the atmosphere depends on the consumption of fossil fuels. As in many other studies, such as Hoel (1993), Wirl (1995), Wirl and Dockner (1995), Tahvonen (1996, 1997), and Rubio and Escriche (2001), this paper assumes that the natural depreciation rate of cumulative emissions is zero, so that emissions are irreversible (In this respect, the cumulative resource extractions are used as a proxy of CO_2 concentration):

$$\dot{S}(t) = a - b(\underbrace{p(t) + \tau(t)}_{\pi(t)}), S(0) = S_0 \geq 0. \tag{1}$$

As one can see, the dynamics of CO_2 concentration will be affected by both the carbon taxation from the consumer side and the (wellhead) energy pricing from the producer side.

Now let us consider the possibility that a carbon-free energy technology which is a perfect substitute for fossil fuels can be invented or discovered at some time in the future. After the innovation or discovery, the new technology can be accessed easily at a constant marginal cost (price) p_N. It is assumed that the cost (price) of the new technology is much lower than that of the fossil energy such that there will be no demand for (or production of) fossil fuels as soon as the innovation in this new technology is made. However, the time of innovation (denoted as t_I) is uncertain. Denote the probability that the new technology has been innovated by time t as $Prob(t_I < t) = H(t)$. Assume for the moment that the hazard rate of the stochastic process leading to the discovery or innovation of the technology is exogenous:

$$\frac{\dot{H}(t)}{1 - H(t)} = \theta, H(0) = 0. \tag{2}$$

The hazard rate θ can be thought of as the (conditional) probability that the new technology will be innovated at time t, given that this has not happened before time t. Of course, we have $\theta \geq 0$, and $\theta = 0$ would represent the case in which no innovation can happen, i.e., the possibility of innovation is zero. The c.d.f. (cumulative distribution function) and p.d.f (probability density function) of the random variable t_I can be obtained from (2) as $H(t) = 1 - e^{-\theta t}$ and $h(t) = \theta e^{-\theta t}$, respectively. As can be seen, parameter θ affects the probability distribution of the time for innovation. This specification for uncertain arrival time of innovation has been employed widely in the previous studies, e.g., Harris and Vickers (1995).

After the innovation of the carbon-free technology, there would be no further emissions, thereby making the CO_2 concentration constant. That is, we actually have:

$$\dot{S}(t) = \begin{cases} a - b(\underbrace{p(t) + \tau(t)}_{\pi(t)}) & if\, t < t_I \\ 0 & if\, t \geq t_I \end{cases} . \tag{3}$$

Taking account of the possible innovation in the new technology, the consumers' coalition wants to maximize the present value of the net consumers' welfare which consists of consumers' surplus plus carbon tax revenues minus the damage cost of climate change, by choosing the carbon tax $\tau(t)$:

$$\max_{\{\tau(t)\}} E \left\{ \int_0^{t_I} e^{-rt}[u(p(t) + \tau(t)) + \tau(t)D(p(t) + \tau(t)) - \Omega(S(t))]dt \right. \\ \left. + \int_{t_I}^{\infty} e^{-rt}[u(p_N) - \Omega(S(t))]dt \right\}, \tag{4}$$

where r is the discount rate, $u(p(t) + \tau(t)) = u(\pi(t)) = \int_{\pi(t)}^{\pi^c} D(x)dx$ is the consumes' surplus, where π^c is the choke price which makes $D(\pi^c) = 0$. With linear demand we have $\pi^c = \frac{a}{b}$ and thus $u(p(t) + \tau(t)) = \frac{1}{2}a\pi^c + \frac{1}{2}b[p(t) + \tau(t)]^2 - a[p(t) + \tau(t)]$. The term $\tau D(p(t) + \tau(t))$ in (4) represents the tax revenues and they are reimbursed to the consumers. Since these tax revenues are not taken into account by the consumers' surplus $u(\cdot)$, they are added explicitly in (4). The external cost of climate change is represented by a quadratic damage function $\Omega(S(t)) = \varepsilon[S(t)]^2$, where $\varepsilon > 0$. The expectation operator $E\{\cdot\}$ appears in (4) due to uncertainty about the time when the innovation of the new technology will occur (i.e., t_I). As mentioned above, the new technology can be accessed easily at a constant marginal cost (price) p_N (which is lower than that of the fossil fules) after the occurrence of innovation, which implies that the consumers' surplus at time $t \geq t_I$ would be a constant $\bar{u} = u(p_N) = \frac{1}{2}a\pi^c + \frac{1}{2}b(p_N)^2 - ap_N$. Since there would be no further fossil energy consumption and emissions with the new technology, the CO_2 concentration will keep constant after the innovation is made, as indicated in (3). But there will still be instantaneous environmental damages coming from the previous emission accumulations due to the irreversibility of emissions, as shown in (4).

With the producers' surplus being neglected by the consumers' coalition, the external cost of climate change is ignored by the energy producer's cartel and it concentrates only on maximizing the present value of its net profits through the (wellhead) energy pricing strategy $p(t)$:

$$\max_{\{p(t)\}} E\left\{\int_0^{t_I} e^{-rt}[(p(t) - cS(t))D(p(t) + \tau(t))]dt\right\}, \tag{5}$$

where $c > 0$ is the ratio of marginal extraction cost to cumulative extraction (the marginal extraction cost will increase linearly with the cumulative extraction). Since there will be no further demand for fossil fuels after the innovation is made, the producers' cartel will receive zero profit after the innovation time t_I. Again, due the uncertainty of innovation time, (5) comes with the expectation operator $E\{\cdot\}$.

As in many other related studies (e.g., Wirl, 1994; Tahvonen, 1994, 1996, 1997; Rubio and Escriche, 2001; Liski and Tahvonen, 2004), the natural resource constraints are ignored, which implies that the cumulative extractions (emissions) are not constrained by the resource in the ground.[1] The strategic interactions between a consumers' coalition and a producers' cartel with possible innovation in a carbon-free technology is thus modeled by a stochastic dynamic game where the time of innovation is uncertain. Since there will be no more fossil energy consumption and carbon taxation after the innovation, the game is essentially ended at a stochastic time t_I when the innovation of the new technology occurs.

2.2 Markov-perfect Nash equilibrium

The stochastic dynamic game developed in Section 2.1 is essentially a piecewise deterministic differential game with two modes (regimes): mode $k = 0$ is active before the innovation of the new technology and mode $k = 1$ becomes active after the new technology is invented (or discovered). After the innovation, the game will stay in mode 1; therefore, there can be at most one switch of mode in the game. The hazard rate of switching is assumed to be exogenous at the moment (i.e., θ is considered as an exogenous parameter in this section and the one that follows) and it will be made endogenous in Section 4.

Compared with an open-loop Nash equilibrium, a Markov-perfect Nash equilibrium would be more interesting in the context of strategic interactions since it provides a subgame perfect equilibrium that is dynamically consistent (Rubio and Escriche,

[1]As stated by Wirl (2007), this assumption emphasizes that the atmosphere as sink instead of the resources in the ground constrains fossil energy use. If fossil fuels are insufficient to raise global temperature significantly, then global warming would not be a serious problem.

2001). Define $W(k, S)$ and $V(k, S)$ as the current value functions for the consumers' coalition and the producers' cartel (respectively) in system mode $k = 0, 1$. The players' Markovian strategies $\tau(k, S)$ and $p(k, S)$ need to satisfy the following Hamilton-Jacobi-Bellman (HJB) equations:

$$\begin{aligned} rW(0,S) = \max_{\{\tau\}} \big\{ & u(p^*(0,S)+\tau) + \tau D(p^*(0,S)+\tau) - \varepsilon S^2 \\ & + D(p^*(0,S)+\tau) W_S(0,S) + \theta[W(1,S) - W(0,S)] \big\}, \end{aligned} \tag{6.1}$$

$$rW(1,S) = \bar{u} - \varepsilon S^2, \tag{6.2}$$

$$\begin{aligned} rV(0,S) = \max_{\{p\}} \{ & (p - cS) D(p + \tau^*(0,S)) \\ & + D(p+\tau^*(0,S)) V_S(0,S) + \theta[V(1,S) - V(0,S)] \}, \end{aligned} \tag{6.3}$$

$$rV(1,S) = 0, \tag{6.4}$$

where $W_S(0, S)$ and $V_S(0, S)$ are the first order derivatives of respective value functions $W(0, S)$ and $V(0, S)$ with respect to the CO_2 concentration level S. HJB equations (6.1) and (6.3) suggest that both players need to take into account the possibility of innovation in the new technology for decision-making if the innovation has not happened yet (system is in mode 0). Equation (6.2) says that the consumers' coalition will receive constant consumers' surplus and suffer from (constant) instantaneous environmental damage after the occurrence of innovation. Since there is no more profits for resource producers after the innovation, equation (6.4) holds. It should be noticed that the carbon taxation and (wellhead) energy pricing decisions need to be made in mode $k = 0$ only, i.e., before the innovation. Therefore, players' Markovian strategies can be denoted as $\tau(0, S)$ and $p(0, S)$, where 0 indicates that the innovation has not yet happened, i.e., the model system is in mode 0.

From the first-order conditions for the maximization of the right-hand sides of the HJB equations (6.1) and (6.3), one can get the consumers and producers' optimal strategies:

$$\tau(0,S) = -W_S(0,S), \tag{7.1}$$

$$p(0,S) = \frac{1}{2}\left[\pi^c + cS + [W_S(0,S) - V_S(0,S)]\right]. \tag{7.2}$$

Consequently one can obtain the equilibrium consumer price by summing up the carbon tax and energy price:

$$\pi(0,S) = \frac{1}{2}\left[\pi^c + cS - [W_S(0,S) + V_S(0,S)]\right]. \tag{7.3}$$

By incorporating the optimal strategies into the HJB equations one can then obtain a pair of differential equations for the value functions. More specifically, substitute the optimal strategies (7.1) and (7.2) together with the value functions $W(1,S)$ in (6.2) and $V(1,S)$ in (6.4) into the HJB equations (6.1) and (6.3) and eliminate the maximization. After some calculations one can obtain the following differential equations:

$$(r+\theta)W(0,S) = \frac{1}{8}b[\pi^c - cS + W_S(0,S) + V_S(0,S)]^2 - (1+\frac{\theta}{r})\varepsilon S^2 + \theta\frac{\bar{u}}{r}, \tag{8.1}$$

$$(r+\theta)V(0,S) \ = \frac{1}{4}b[\pi^c - cS + W_S(0,S) + V_S(0,S)]^2. \tag{8.2}$$

Due to the linear-quadratic structure of the game, let us conjecture quadratic forms for the value functions $W(0,S)$ and $V(0,S)$. That is:

$$W(0,S) = w_0 + w_1 S + \frac{1}{2}w_2 S^2,\ V(0,S) = v_0 + v_1 S + \frac{1}{2}v_2 S^2, \tag{9}$$

where w_0, w_1, w_2, v_0, v_1, and v_2 are coefficients to be determined. Substituting (9) into

(8.1) and (8.2) and collecting items we have:

$$
\begin{aligned}
(r+\theta)[w_0 + w_1 S + \frac{1}{2} w_2 S^2] &= \frac{1}{8} b[\pi^c + w_1 + v_1 + (w_2 + v_2 - c)S]^2 \\
&\quad - (1+\frac{\theta}{r})\varepsilon S^2 + \theta \frac{\bar{u}}{r},
\end{aligned} \tag{10.1}
$$

$$
(r+\theta)[v_0 + v_1 S + \frac{1}{2} v_2 S^2] \ = \frac{1}{4} b[\pi^c + w_1 + v_1 + (w_2 + v_2 - c)S]^2. \tag{10.2}
$$

Equating the coefficients of 1, S and S^2 on the two sides of (10.1) and (10.2) leads to a equation system consisting of 6 equations. Solving the equation system for w_i and v_i, $i = 0, 1, 2$ (one trick is to define new variables $z = w_2 + v_2$, $x = w_1 + v_1$), one can obtain the coefficients for the value functions $W(0, S)$ and $V(0, S)$, as shown in Table 1, where

$$
z = w_2 + v_2 = c + \frac{2}{3b}\left(r + \theta - \sqrt{(r+\theta)^2 + 3(r+\theta)bc + 6b\varepsilon(1+\frac{\theta}{r})}\right), \tag{11.1}
$$

$$
x = w_1 + v_1 = \frac{4(r+\theta)\pi^c}{4(r+\theta) + 3b(c-z)} - \pi^c, \tag{11.2}
$$

and one can verify that $c - z > 0$ and $x < 0$.[2]

Based on the value functions (9) and their coefficients in Table 1, one can obtain the equilibrium strategies of the consumers' coalition and the producers' cartel as functions of model parameters and CO_2 concentration level by substituting the value functions (9) into the equilibrium strategies (7.1) and (7.2):

$$
\tau(0, S) = -w_1 - w_2 S, \tag{12.1}
$$

$$
p(0, S) = \frac{1}{2}\left[\pi^c + (w_1 - v_1) + [c + (w_2 - v_2)]S\right], \tag{12.2}
$$

where w_1, w_2, v_1 and v_2 are the coefficients of the value functions as in Table 1. The

[2]The procedure for calculating the coefficients is similar to the one that is used by Wirl and Dockner (1995) and Rubio and Escriche (2001). Therefore, the detailed procedure is omitted. Complete computation is available upon request.

equilibrium consumer price can be obtained by summing up (12.1) and (12.2):

$$\pi(0,S) = \tau(0,S) + p(0,S) = \frac{1}{2}\left[\pi^c - (w_1 + v_1) + [c - (w_2 + v_2)]S\right]. \tag{12.3}$$

Table 1. Coefficients for value functions $W(0,S)$ and $V(0,S)$

$w_0 = \frac{b}{8(r+\theta)}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)}\right]^2 + \frac{\theta\bar{u}}{r(r+\theta)}$	$v_0 = \frac{b}{4(r+\theta)}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)}\right]^2$
$w_1 = \frac{1}{3}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)} - \pi^c\right] = \frac{1}{3}x$	$v_1 = \frac{2}{3}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)} - \pi^c\right] = \frac{2}{3}x$
$w_2 = \frac{1}{3}\left[z - \frac{4\varepsilon}{r}\right]$	$v_2 = \frac{2}{3}\left[z + \frac{2\varepsilon}{r}\right]$

Plugging (12.3) into the differential equation (1) and solving the equation, one can find the temporal trajectory for CO_2 concentration before the innovation:

$$S(t) = S_\infty + (S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\} \quad if\, t < t_I, \tag{13}$$

where S_0 is the initial CO_2 concentration (cumulative emissions) and S_∞ is the long-run CO_2 concentration equilibrium or steady state for system mode 0, (i.e., before the innovation), which can be further calculated as:

$$S_\infty = \frac{x + \pi^c}{c - z} = \frac{r\pi^c}{rc + 2\varepsilon}, \tag{14}$$

where (11.1) and (11.2) are made use of for the last equality in (14). It can be seen that the long-run CO_2 concentration equilibrium S_∞ is independent of θ, which implies the that the CO_2 concentration with the possibility of technological innovation (but has not happened yet) would tend to approach the same long-run equilibrium CO_2 concentration as in the case where no innovation can happen (i.e., $\theta = 0$).[3] Due to

[3] This is due to the fact that as time goes to infinity, the uncertainty of innovation tends to vanish.

the assumption of irreversible emissions, it is reasonable to have $S_0 < S_\infty$. Therefore, it can be seen from (13) that the CO_2 concentration (cumulative emissions) before the occurrence of innovation (in mode 0) will increase monotonically toward the long-run equilibrium level S_∞ (recall $c - z > 0$).

Plugging (13) into the equilibrium strategies (12.1)-(12.3), one can obtain the temporal trajectories of carbon tax, producer price and consumer price (before the occurrence of innovation, i.e., in mode 0) after some calculations[4]:

$$\tau(0,t) = \frac{2\pi^c\varepsilon}{rc+2\varepsilon} - w_2(S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\}, \tag{15.1}$$

$$p(0,t) = \frac{c\pi^c r}{rc+2\varepsilon} + \frac{1}{2}(c + w_2 - v_2)(S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\}, \tag{15.2}$$

$$\pi(0,t) = \pi^c + \frac{1}{2}(c-z)(S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\}. \tag{15.3}$$

It can be observed from these equations that, if the innovation has not yet happened(in system mode 0), the equilibrium strategies of carbon taxation and (wellhead) energy pricing would follow the paths towards long-run equilibria that are characterized by $\tau_\infty = \dfrac{2\pi^c\varepsilon}{rc+2\varepsilon}$ and $p_\infty = \dfrac{c\pi^c r}{rc+2\varepsilon}$, respectively. The equilibrium consumer price will approach the choke price π^c in the long run, i.e., $\pi_\infty = \pi^c$. Moreover, it is noticeable that the long-run equilibria τ_∞, p_∞, and π_∞ are independent of the hazard rate of innovation θ. This implies that, with the possibility of technological innovation (that has not happened yet), the long-run equilibrium carbon tax, (producer) energy price, and consumer price would be the same as those in the case without the possibility of innovation ($\theta = 0$). It can also be observed from (15.3) that the equilibrium consumer price would increase monotonically over time (recall $c - z > 0$ and $S_0 < S_\infty$). However, how the carbon tax and producer price would evolve over time

[4]The calculations are omitted here to save space. Complete computation is available upon request.

is still ambiguous. To see this, recall $w_2 = \frac{1}{3}[z - \frac{4\varepsilon}{r}]$ and $v_2 = \frac{2}{3}[z + \frac{2\varepsilon}{r}]$, where we have $z = c + \frac{2}{3b}\big(r + \theta - \sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}\big)$ (see (11.1)). By varying values of c and ε, we could make w_2 either negative or positive. Similarly, the sign of $c + w_2 - v_2$ will also depend on the relative magnitude of c and ε (keeping other parameters constant). This implies that the slopes of temporal trajectories of carbon tax and producer price are ambiguous. However, since the consumer price is increasing, we know that at least one of the two (carbon tax or producer price) needs to be increasing over time. That is, only three cases are possible: (i) increasing carbon tax and decreasing producer price; (ii) decreasing carbon tax and increasing producer price; (iii) both the carbon tax and producer price are increasing. An economic interpretation for this can be stated in terms of whether the increase in extraction cost dominates the increase in environmental damages, or the other way around (Wirl 1995; Rubio and Escriche, 2001). For instance, if the environmental damage is high enough, we will have increasing carbon tax and decreasing producer/wellhead price (it can be verified that $w_2 \to -\infty$ and $c + w_2 - v_2 \to -\infty$ if $\varepsilon \to +\infty$, where it should be kept in mind that the order of infinity will be lowered by a square root).

2.3 Cooperative strategies

The Markov-perfect Nash equilibrium obtained above is based on the assumption that the two players have conflicting objectives: the consumers' coalition cares about the consumers' welfare and damage of climate change while the producers' cartel cares only about the profits from resource extraction. In this section, the cooperative solution, i.e., the global efficient strategy, for the dynamic game will be calculated and investigated. As stated by Wirl (1995), this efficient strategy can serve as the benchmark and provide more insights into the strategic interaction issues by comparing the global efficient solution with the non-cooperative solution (Markov-perfect Nash equilibrium).

It should be noticed that, in the cooperative case, the consumers' welfare and the producers' profits needs to be added together to account for the global welfare. That is:

$$E\left\{\int_0^{t_I} e^{-rt}[u(p(t)+\tau(t))+(p(t)+\tau(t)-cS(t))D(p(t)+\tau(t))-\Omega(S(t))]dt \right. \\ \left. +\int_{t_I}^{\infty} e^{-rt}[u(p_N)-\Omega(S(t))]dt\right\}. \tag{16}$$

In the cooperative case, the maximization of global welfare (16) is by definition the same for consumers' coalition and producers' cartel so that the split of the consumer price π into a producer price p and the carbon tax τ is indefinite, thereby making the final consumer price π become the only decision variable in the maximization of (16) (i.e., $p(t)+\tau(t)$ can be replaced by $\pi(t)$ in (16)):

$$\max_{\{\pi(t)\}} E\left\{\int_0^{t_I} e^{-rt}[u(\pi(t))+(\pi(t)-cS(t))D(\pi(t))-\Omega(S(t))]dt \right. \\ \left. +\int_{t_I}^{\infty} e^{-rt}[u(p_N)-\Omega(S(t))]dt\right\}. \tag{17}$$

Similar to the non-cooperative case in Section 2.2, define $M(k,S)$ as the current value functions for the global planner associated with (17) in system mode k. Then the global efficient/optimal strategy needs to satisfy the following HJB equations:

$$rM(0,S) = \max_{\{\pi\}}\{u(\pi)+(\pi-cS)D(\pi)-\varepsilon S^2+D(\pi)M_S(0,S) \\ +\theta[M(1,S)-M(0,S)]\}, \tag{18.1}$$

$$rM(1,S) = \bar{u}-\varepsilon S^2, \tag{18.2}$$

where $M_S(0,S)$ is the first order derivative of value function $M(0,S)$ with respect to the CO_2 concentration S.

The first-order condition for the maximization of the right-hand sides of the HJB equations (18.1) gives the global efficient strategy:

$$\pi^G(0,S) = -M_S(0,S) + cS. \tag{19}$$

Substitute the optimal strategy (19) and the value functions $M(1,S)$ from (18.2) into the HJB equations (18.1), eliminate the maximization, and, after some calculations, we have

$$(r+\theta)M(0,S) = \frac{1}{2}b[\pi^c - cS + M_S(0,S)]^2 - (1+\frac{\theta}{r})\varepsilon S^2 + \theta\frac{\bar{u}}{r}. \tag{20}$$

Conjecture quadratic value function

$$M(0,S) = m_0 + m_1 S + \frac{1}{2}m_2 S^2, \tag{21}$$

where m_0, m_1, and m_2 are coefficients that need to be determined. Substituting (21) into (20) and collecting items, we have:

$$(r+\theta)[m_0 + m_1 S + \frac{1}{2}m_2 S^2] = \frac{1}{2}b\,[\pi^c + m_1 + (m_2 - c)S]^2 - (1+\frac{\theta}{r})\varepsilon S^2 + \theta\frac{\bar{u}}{r}. \tag{22}$$

Equating the coefficients of 1, S, and S^2 on the two sides of (22) and solving for m_0, m_1, and m_2, one can obtain the coefficients for the value function $M(0,S)$, as shown in Table 2.[5] It can be verified that $c - m_2 > 0$.

Substituting the value functions (21) with the calculated coefficients, one can obtain the global efficient strategy as a function of model parameters and CO_2 concentration

[5] We have omitted the detailed calculations. They are available from the author upon request.

level:

$$\pi^G(0,S) = (c - m_2)S - m_1, \tag{23}$$

where $m_1$and $m_2$are the coefficients of the value functions as in Table 2. Plugging (23) into the differential equation (1) and solving the equation, one can get the explicit solution:

$$S^G(t) = S_\infty^G + (S_0 - S_\infty^G)\exp\{-b(c-m_2)t\} \quad if\, t < t_I, \tag{24}$$

where S_0 is the initial CO_2 concentration (cumulative emissions) and $S_\infty^G = \dfrac{\pi^c + m_1}{c - m_2} = \dfrac{r\pi^c}{rc + 2\varepsilon}$ is the long-run CO_2 concentration equilibrium or steady state for system mode 0, i.e., before the innovation.

Table 2. Coefficients for value function $M(0,S)$

$m_0 = \dfrac{b}{2(r+\theta)}\left[\dfrac{(r+\theta)\pi^c}{(r+\theta)+b(c-m_2)}\right]^2 + \dfrac{\theta\bar{u}}{r(r+\theta)}$
$m_1 = \dfrac{(r+\theta)\pi^c}{(r+\theta)+b(c-m_2)} - \pi^c$
$m_2 = c + \dfrac{1}{2b}\left(r + \theta - \sqrt{(r+\theta)^2 + 4bc(r+\theta) + 8b\varepsilon(1+\frac{\theta}{r})}\right)$

It can be noticed that the long-run CO_2 concentration equilibrium in the cooperative case is the same as that in the non-cooperative case, i.e., $S_\infty^G = S_\infty$. Similar to the non-cooperative case, the long-run CO_2 concentration S_∞^G is also independent of θ, the hazard rate of innovation. Besides, one can see from (24) that the CO_2 concentration under the global efficient strategy will also increase monotonically before the occurrence of innovation (in mode 0) toward the long-run equilibrium level S_∞^G (recall $c - m_2 > 0$).

Plugging (24) into (23), one can obtain the temporal trajectory of the global efficient strategy (before the occurrence of innovation, i.e., in mode 0) after some calculations:

$$\pi^G(0,t) = \pi^c + (c - m_2)(S_0 - S_\infty^G)\exp\{-b(c - m_2)t\}\,. \tag{25}$$

It can also be observed from (25) that the equilibrium consumer price in the cooperative case would also increase monotonically over time (since $c - m_2 > 0$ and $S_0 < S_\infty^G$).

2.4 Comparison of the cooperative and non-cooperative solutions

While the global efficient strategy serves as a benchmark or first-best solution for the global warming problem, the non-cooperative solution reflects the effect of strategic interactions between the consumers' coalition and the producers' cartel. Therefore, it would be of interest to compare the cooperative and non-cooperative solutions. Specifically, by comparing the consumer price in the cooperative case with that in the non-cooperative case, one can find the result summarized in Proposition 1

Proposition 1 *The consumer price in the global efficient solution has a lower initial value than that in the Markov-perfect Nash equilibrium.*

Proof. Recall from (15.3) and (25) that the temporal trajectories of consumer prices in the cooperative and non-cooperative cases are, respectively:

$$\pi^G(0,t) = \pi^c + (c - m_2)(S_0 - S_\infty^G)\exp\{-b(c - m_2)t\}\,,$$

$$\pi(0,t) = \pi^c + \frac{1}{2}(c - z)(S_0 - S_\infty)\exp\{-\frac{1}{2}b(c - z)t\}\,.$$

Note that the initial consumer prices in the two cases are, respectively:

$$\pi^G(0,0) = \pi^c + (c - m_2)(S_0 - S_\infty^G),$$

$$\pi(0,0) = \pi^c + \frac{1}{2}(c-z)(S_0 - S_\infty),$$

where we have (from (11.1) and Table 2)

$$\frac{1}{2}(c-z) = -\frac{1}{3b}\left(r+\theta - \sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}\right),$$

$$c - m_2 = -\frac{1}{2b}\left(r+\theta - \sqrt{(r+\theta)^2 + 4bc(r+\theta) + 8b\varepsilon(1+\frac{\theta}{r})}\right).$$

As mentioned before, both $\frac{1}{2}(c-z)$ and $c - m_2$ are positive. If we can know the sign of $(c-m_2) - \frac{1}{2}(c-z)$, we can say something about the comparison of initial consumer prices in the cooperative and non-cooperative cases. Since we have

$$(c-m_2) - \frac{1}{2}(c-z) = -\frac{1}{6b}(r+\theta) + \frac{1}{2b}\sqrt{(r+\theta)^2 + 4bc(r+\theta) + 8b\varepsilon(1+\frac{\theta}{r})}$$
$$- \frac{1}{3b}\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}$$

and we know $\sqrt{(r+\theta)^2 + 4bc(r+\theta) + 8b\varepsilon(1+\frac{\theta}{r})} > \sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}$, this implies $(c-m_2) - \frac{1}{2}(c-z) > -\frac{1}{6b}(r+\theta) + \frac{1}{6b}\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})} > 0$. Therefore, we have: $\pi^G(0,0) < \pi(0,0)$, i.e., the initial consumer price is lower for the cooperative case. ■

This implies that the strategic interaction or rent contest between the consumers' coalition and the producers' cartel will decrease the initial fossil fuel consumption, compared with the case when they are cooperating with each other. However, it should be noticed that consumer prices in both the competitive and cooperative cases will approach the same steady level π^c in system mode $k = 0$, as indicated by (15.3) and (25).

3 The effect of possible innovation

Based on the game's cooperative and non-cooperative solutions obtained above, one can analyze the effect of possible innovation on both solutions by comparing the case with a positive θ (with possible innovation) and the case of $\theta = 0$ (with no innovation). It should be noticed that since the dynamic game is essentially ended after the innovation, the analysis will be concentrated on the effect of possible innovation in system mode $k = 0$, in which the innovation has not yet happened but the players expect that it can happen sometime in the future.

3.1 Effect of possible innovation on the Markov-perfect Nash equilibrium

To see the effect of possible innovation on the non-cooperative strategies, let us take the derivatives of (15.1)-(15.3) with respect to the hazard rate of innovation θ. After some calculations one can obtain:

$$\frac{\partial \tau(0,t)}{\partial \theta} = -\frac{1}{6}\frac{\partial z}{\partial \theta}(2 + 3bw_2 t)(S_0 - S_\infty) \exp\left\{-\frac{1}{2}b(c-z)t\right\}, \tag{26.1}$$

$$\frac{\partial p(0,t)}{\partial \theta} = -\frac{1}{12}\frac{\partial z}{\partial \theta}\left[2 - 3b(c + w_2 - v_2)t\right](S_0 - S_\infty) \exp\left\{-\frac{1}{2}b(c-z)t\right\}, \tag{26.2}$$

$$\frac{\partial \pi(0,t)}{\partial \theta} = -\frac{1}{4}\frac{\partial z}{\partial \theta}[2 - b(c-z)t](S_0 - S_\infty) \exp\left\{-\frac{1}{2}b(c-z)t\right\}. \tag{26.3}$$

First, let us find out how the possible innovation will affect the initial carbon tax, producer price and consumer price. The results are summarized in the following proposition.

Proposition 2 *The possible innovation in the new technology will lead to a lower initial carbon tax and producer price, thereby resulting in a lower initial consumer price.*

Proof. By substituting $t = 0$ into (26.1)-(26.3), one can obtain the marginal effect of innovation hazard rate on the initial values of carbon tax, fuel price, and consumer price:

$$\frac{\partial \tau(0,0)}{\partial \theta} = -\frac{1}{3}\frac{\partial z}{\partial \theta}(S_0 - S_\infty), \tag{27.1}$$
$$\frac{\partial p(0,0)}{\partial \theta} = -\frac{1}{6}\frac{\partial z}{\partial \theta}(S_0 - S_\infty), \tag{27.2}$$
$$\frac{\partial \pi(0,0)}{\partial \theta} = -\frac{1}{2}\frac{\partial z}{\partial \theta}(S_0 - S_\infty). \tag{27.3}$$

Since $S_0 < S_\infty$, one can identify the signs of (27.1)-(27.3) if the sign of $\frac{\partial z}{\partial \theta}$ is known. It has been shown in Appendix A1 $\frac{\partial z}{\partial \theta} < 0$ for all $\theta \geq 0$. Thus, we have $\frac{\partial \tau(0,0)}{\partial \theta} < 0$, $\frac{\partial p(0,0)}{\partial \theta} < 0$, and $\frac{\partial \pi(0,0)}{\partial \theta} < 0$, which implies $\tau(0,0)|_{\theta>0} < \tau(0,0)|_{\theta=0}$, $p(0,0)|_{\theta>0} < p(0,0)|_{\theta=0}$ and $\pi(0,0)|_{\theta>0} < \pi(0,0)|_{\theta=0}$. That is, the anticipation of possible innovation will lower the initial carbon taxation, fuel price, and consumer price. ■

This result suggests that the possibility of innovation will stimulate a higher initial demand for fossil fuels, and thus higher initial emissions. With the expectation that the innovation of a carbon-free technology can happen and will relieve the concerns about environmental damage, the consumers' coalition lowers the initial carbon tax. Being aware that occurrence of innovation would lead to zero demand for the fossil energy, the producers' cartel also would like to lower the initial (wellhead) energy price to stimulate the consumption of fossil fuels. Consequently, the initial consumer price is lower as a result of the reduced carbon tax and producer price, and this leads to a higher initial demand for fossil fuels and higher initial CO_2 emissions.

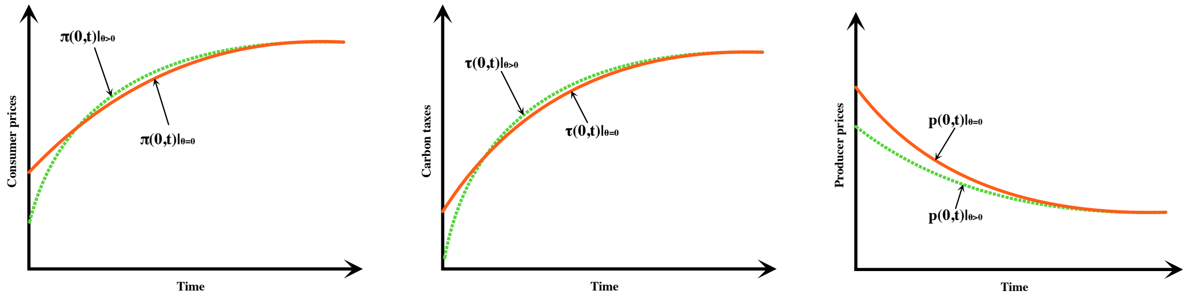
As can be seen in (15.1)-(15.3), due to the uncertainty about the time of innovation, a positive probability of innovation ($\theta > 0$) will not change the long-run equilibrium of carbon tax, producer price and consumer price before the innovation (i.e., in system mode $k = 0$). However, the possible innovation will affect the transitional dynamics of these variables in addition to their initial values. If we denote the case with possible innovation ($\theta > 0$) as case WPI and the case with no innovation ($\theta = 0$) as case WNI, respectively, then the following proposition can be established and demonstrated.

Proposition 3 *Provided that the model system is in mode $k = 0$, we have the following statements: (i) The consumer price in WPI is first below, but later above the consumer price in WNI; (ii) If the environmental damage is sufficiently high, the carbon tax in WPI will be first lower, but later higher than the carbon tax in WNI; (iii) The producer price in WPI will always be lower than that in WNI, if the environmental damage is high enough.*

Proof. Recall from (26.3) that the derivative of consumer price (in mode 0) w.r.t. the hazard rate of innovation is calculated as:

$$\frac{\partial \pi(0,t)}{\partial \theta} = -\frac{1}{4}\frac{\partial z}{\partial \theta}[2 - b(c-z)t](S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\}.$$

It has been shown in Proposition 2 that, for $t = 0$, the effect of innovation possibility on the consumer price (i.e.,$\frac{\partial \pi(0,0)}{\partial \theta}$) is negative. For $t \neq 0$, since $c - z > 0$, we can find a $t^* > 0$ to make $2 - b(c-z)t^* = 0$. Recall that $\frac{\partial z}{\partial \theta} < 0$(see Appendix A1), and thus we have $\frac{\partial \pi(0,t)}{\partial \theta} < 0$ for $0 \leq t < t^*$ and $\frac{\partial \pi(0,t)}{\partial \theta} > 0$ for $t > t^*$. This implies that, for $0 \leq t < t^*$, the consumer price with the expectation of possible innovation would be lower than that in the case without such an expectation (i.e., $\pi(0,t)|_{\theta>0} < \pi(0,t)|_{\theta=0}$) , whereas for $t > t^*$ the relationship is the contrary. Figure 1(a) illustrates this result.



a. Consumer price b. Carbon tax c. Producer price

Figure 1. Temporary trajectories for players' strategies with/without the innovation possibility

Similarly, for the carbon tax we have from (26.1) that:

$$\frac{\partial \tau(0,t)}{\partial \theta} = -\frac{1}{6}\frac{\partial z}{\partial \theta}(2+3w_2bt)(S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\}.$$

Since $\frac{\partial z}{\partial \varepsilon} = \frac{-2(1+\frac{\theta}{r})}{\sqrt{(r+\theta)^2+3bc(r+\theta)+6b\varepsilon(1+\frac{\theta}{r})}} < 0$ and $w_2 = \frac{1}{3}\left[z - \frac{4\varepsilon}{r}\right]$, we have $\frac{\partial w_2}{\partial \varepsilon} = \frac{1}{3}\left[\frac{\partial z}{\partial \varepsilon} - \frac{4}{r}\right] < 0$. Besides, it can be found that $z \to -\infty$, thus $w_2 \to -\infty$ if $\varepsilon \to +\infty$. That is, if the environmental damage ε is high enough, we have $w_2 < 0$, which implies that we can find a $t^{**} > 0$ which satisfies $2 + 3w_2bt^{**} = 0$. Thus we have $\frac{\partial \tau(0,t)}{\partial \theta} < 0$ for $0 \leq t < t^{**}$ and $\frac{\partial \tau(0,t)}{\partial \theta} > 0$ for $t > t^{**}$, which implies $\tau(0,t)|_{\theta>0} < \tau(0,t)|_{\theta=0}$ for $0 \leq t < t^{**}$ and $\tau(0,t)|_{\theta>0} > \tau(0,t)|_{\theta=0}$ for $t > t^{**}$. Therefore, if the environmental damage is high enough, the carbon tax with possible innovation will be first below, but later above, the carbon tax in the case with no innovation, as illustrated in Figure 1(b).

As for the producer price, we have from (26.2) that:

$$\frac{\partial p(0,t)}{\partial \theta} = -\frac{1}{12}\frac{\partial z}{\partial \theta}\left[2 - 3b(c + w_2 - v_2)t\right](S_0 - S_\infty)\exp\left\{-\frac{1}{2}b(c-z)t\right\}.$$

Recall from Table 1 that $w_2 - v_2 = -\frac{1}{3}[z + \frac{8\varepsilon}{r}]$. If $\varepsilon \to +\infty$, we have $z \to -\infty$ and $\frac{8\varepsilon}{r} \to +\infty$. However, the order of infinity is lower for z because of the square root. Therefore, we have $z + \frac{8\varepsilon}{r} \to +\infty$, and thus $w_2 - v_2 \to -\infty$, if $\varepsilon \to +\infty$. That is, if the environmental damage ε is high enough, we can have $c + w_2 - v_2 < 0$, which implies that we will have $[2 - 3b(c + w_2 - v_2)t] > 0$ and thus $\frac{\partial p(0,t)}{\partial \theta} < 0$ for all $t \geq 0$. That is, if the environmental damage is high enough, the producer price with the expectation of possible innovation will be lower than that in the case without such an expectation (i.e., $p(0,t)|_{\theta>0} < p(0,t)|_{\theta=0}$) before the producer price for the two cases converge to the same long-run equilibrium (recall p_∞ is independent of the hazard rate of innovation θ). Figure 1(c) provides an illustration. ■

These results are reflective of the fact that the CO_2 concentration in both cases (namely, case WPI with mode 0, and case WNI) will converge to the same long-run equilibrium level, which implies that the total amount of fossil energy consumed in both cases is the same and the area under the temporal path of fossil energy demand should also be the same. This implies that the area under the temporal path of the consumer price (which determines the fossil energy demand) would be the same for both cases as well. Since we have shown in Proposition 2 that the initial consumer price will be lower with the possible innovation (compared with the case of no innovation), the temporal paths of the consumer price in the two cases need to intersect to get the same area under the temporal path. The monotonic property of consumer price implies that the paths in the two cases intersect only one.

As mentioned above in Section 2, with strategic interactions between energy consumers and producers, the carbon tax will be increasing over time and the (wellhead) fuel price will be decreasing if the environmental damage is sufficiently high. Given that the consumer price is always increasing over time, the proportion of (wellhead) fuel price in the consumer price would be decreasing if the damage is high enough, which implies that, if the environmental damage is sufficiently high, the temporal trajectory of consumer price will depend mainly on that of the carbon tax. Therefore, the temporal paths of the carbon taxes need to intersect such that the temporal paths of the consumer price can intersect. Since the initial carbon tax is lower for the case with possible innovation than the case without, as shown in Proposition 2, we will see that the carbon tax with the expectation of possible innovation would be first below, but later above, the carbon tax without such an expectation. The intersection of the temporary paths for carbon taxes and the decreasing proportion of producer price in the consumer price can leave room for the (wellhead) fuel price in the two cases (with and without possible innovation) not to intersect.

It should be emphasized that Proposition 3 is established based on the underlying assumption that the model system is still in mode 0, i.e., even though the innovation can happen (in the case of $\theta > 0$), it has not happened yet. Since the time of innovation is uncertain, it can happen at any time. If the innovation occurs at some time that is earlier than the critical time t^* or t^{**}, the conclusions in Proposition 3 should be modified accordingly, given that the occurrence of innovation will bring cheap non-polluting technology. For instance, if the innovation time $t_I < t^{**}$, the carbon tax in the case of $\theta > 0$ may not have the chance to be higher than that in the case of $\theta = 0$, given that carbon tax will be zero after the innovation.

One of the implications from Proposition 3 is that the fossil fuel demand (thus CO_2 emissions) in the case with possible innovation will be first above, but later below, the demand (and thus emissions) in the case with no innovation. Since the temporal paths of CO_2 concentration (cumulative emissions) in both cases are monotonically increasing over time and have the same initial and long-run equilibrium level (recall that S_∞ is independent of θ), one can expect that the CO_2 concentration level in the case with the expectation of possible innovation will be above the concentration level in the case without such an expectation for any instant of time $t \in (0, t_I)$, as formally demonstrated in the following proposition.

Proposition 4 *For any instant of time $t \in (0, t_I)$, the CO_2 concentration with the expectation of possible innovation is higher than that in the case without such an expectation.*

Proof. Recall that the evolution of CO_2 concentration level (before the occurrence of innovation) along the equilibrium path is characterized by (13):

$$S(t) = S_\infty + (S_0 - S_\infty) \exp\left\{-\frac{1}{2}b(c-z)t\right\} \quad if\, t < t_I,$$

where $S_\infty = \dfrac{r\pi^c}{rc + 2\varepsilon}$ is the long-run equilibrium concentration level (in mode 0). As claimed in Section 2, since S_∞ is independent of the hazard rate of innovation θ, the long-run equilibrium CO_2 concentration level with the possibility of innovation will be the same as that in the case where there is no possibility of innovation.

However, the possible innovation will have an effect on the temporal trajectory of CO_2 concentration (from the initial level) to reach the long-run equilibrium level. To see this, take the derivatives of the $S(t)$ with respect to the hazard rate of innovation θ and one can obtain:

$$\frac{\partial S(t)}{\partial \theta} = (S_0 - S_\infty) \exp\left\{-\frac{1}{2}b(c-z)t\right\} (\frac{1}{2}b\frac{\partial z}{\partial \theta}t) \quad if\, t < t_I.$$

Given $S_0 < S_\infty$ and the negative sign of $\frac{\partial z}{\partial \theta}$ (see Appendix A1), one can know that $\frac{\partial S(t)}{\partial \theta} > 0$ will hold for any instant of time $t \in (0, t_I)$, which implies $S(t)|_{\theta>0} > S(t)|_{\theta=0}$ for $t \in (0, t_I)$. That is, for $t \in (0, t_I)$, the CO_2 concentration in the case with possible innovation will be higher than that in the case with no innovation. ■

This result suggests that the expectation of possible innovation will lead to a higher transitional CO_2 concentration before the innovation, which reflects the fact that the fossil fuel demand in early days is higher in the case with possible innovation, compared with the case with no innovation. In this respect, the possible innovation plays a role similar to that of a larger discount rate in the consumption of fossil fuels: consume more in the early days and less in the latter days.

3.2 Effect of possible innovation on the global efficient strategy

In addition to the effect of possible innovation on the non-cooperative solution, one would also like to see the effect of innovation in the case of cooperation between the two players. To see this, one can calculate the derivatives of global efficient strategy (25) with respect to the hazard rate of innovation θ as:

$$\frac{\partial \pi^G(0,t)}{\partial \theta} = -\frac{\partial m_2}{\partial \theta}[1 - b(c - m_2)t](S_0 - S_\infty) \exp\{-b(c - m_2)t\}, \tag{28.1}$$

where we made use of the fact $S^G_\infty = S_\infty$. For $t = 0$, we have

$$\frac{\partial \pi^G(0,0)}{\partial \theta} = -\frac{\partial m_2}{\partial \theta}(S_0 - S_\infty). \tag{28.2}$$

In Appendix B1, we show that $\frac{\partial m_2}{\partial \theta} < 0$ for all $\theta \geq 0$. Therefore, given $S_0 < S_\infty$ we have $\frac{\partial \pi^G(0,0)}{\partial \theta} < 0$ for $\theta \geq 0$, which implies that $\pi^G(0,0)|_{\theta>0} < \pi^G(0,0)|_{\theta=0}$. That is, a positive probability of innovation will lower the initial consumer price in the global efficient solution as well, which is consistent with the effect of innovation in the Markov-perfect Nash equilibrium summarized in Proposition 2.

For $t \neq 0$, since $c - m_2 > 0$, similar arguments in Proposition 3 can be applied here. That is, we can find a $t^{***} > 0$ to make $1 - b(c - m_2)t^{***} = 0$. Recall that $\frac{\partial m_2}{\partial \theta} < 0$ (see Appendix B1), and thus we have $\frac{\partial \pi^G(0,t)}{\partial \theta} < 0$ for $0 \leq t < t^{***}$ and $\frac{\partial \pi^G(0,t)}{\partial \theta} > 0$ for $t >$

t^{***}, which implies that, similar to the non-cooperative solution, the consumer price in the cooperative solution would also be lower with possible innovation (that has not happened yet) than in the case with no innovation (i.e., $\pi^G(0,t)|_{\theta>0} < \pi^G(0,t)|_{\theta=0}$) for $0 \le t < t^{***}$, whereas the relationship is the contrary for $t \ge t^{***}$.

As for the effect of innovation on the dynamics of CO_2 concentration in the cooperative case, since we have: $S^G(t) = S_\infty + (S_0 - S_\infty)\exp\{-b(c-m_2)t\} \;\; if\, t < t_I$ (see (24) and note that $S^G_\infty = S_\infty$), we can get:

$$\frac{\partial S^G(t)}{\partial \theta} = (S_0 - S_\infty)\exp\{-b(c-m_2)t\}\,(b\frac{\partial m_2}{\partial \theta}t) \quad if\, t < t_I. \tag{28.3}$$

Given $S_0 < S_\infty$ and the negative sign of $\frac{\partial m_2}{\partial \theta}$ (see Appendix B1), one can know from (28.3) that $\frac{\partial S^G(t)}{\partial \theta} > 0$ will hold for any $t \in (0, t_I)$, which implies that $S^G(t)|_{\theta>0} > S^G(t)|_{\theta=0}$ will hold for $t \in (0, t_I)$. That is, in the cooperative case, the possibility of innovation will also lead to a higher transitional CO_2 concentration.

Therefore, it can be seen that the effect of possible innovation in the cooperative case is consistent with that in the non-cooperative case. That is, the possibility of innovation will lower the initial consumer price. And the consumer price with the expectation of possible innovation (that has not happened yet) will first be lower but later higher than the consumer price without the possibility of innovation.

3.3 Comparison of effects in the two cases

The result that the possible innovation will lower the initial consumer price in both the non-cooperative case and the cooperative case implies that the expectation of possible innovation will stimulate higher near-term fossil fuel consumption, and thus higher near-term CO_2 emissions, no matter whether the fossil-fuel consuming countries compete or cooperate with the producing countries. But will the magnitude of such an effect be different in the two (cooperative and non-cooperative) cases? To investigate this question, some further calculations are necessary.

By comparing (28.2) with (26.3), we have:

$$\frac{\partial \pi^G(0,0)}{\partial \theta} - \frac{\partial \pi(0,0)}{\partial \theta} = \left(\frac{1}{2}\frac{\partial z}{\partial \theta} - \frac{\partial m_2}{\partial \theta}\right)(S_0 - S_\infty), \tag{29}$$

Based on the expressions for $\frac{\partial z}{\partial \theta}$ and $\frac{\partial m_2}{\partial \theta}$, it is not difficult to show that, as $\varepsilon \to +\infty$, we have $\frac{1}{2}\frac{\partial z}{\partial \theta} - \frac{\partial m_2}{\partial \theta} \to \sqrt{\varepsilon} \cdot sign\left(\frac{b}{r\sqrt{2b(1+\frac{\theta}{r})}} - \frac{b}{r\sqrt{6b(1+\frac{\theta}{r})}}\right) \to +\infty$. This implies that, if the environment damage is high enough, we can get $\frac{1}{2}\frac{\partial z}{\partial \theta} - \frac{\partial m_2}{\partial \theta} > 0$, thereby making $\frac{\partial \pi^G(0,0)}{\partial \theta} - \frac{\partial \pi(0,0)}{\partial \theta} < 0$. Together with the above-demonstrated results $\frac{\partial \pi^G(0,0)}{\partial \theta} < 0$ and $\frac{\partial \pi(0,0)}{\partial \theta} < 0$, we know that the decrease in the initial consumer price due to the possible innovation is more remarkable in the cooperative case, which implies that the increase in the initial fossil fuel consumption (or equivalently, initial CO_2 emissions) as a response to the possible innovation can be more dramatic in the global efficient solution than that in the Markov-perfect Nash equilibrium, if the environment damage is sufficiently high.

The increase in the initial emissions due to the possible innovation in cheap carbon-free technology is consistent with the 'green paradox' argument in the literature. That is, some climate policies designed to abate carbon emissions might actually increase the emissions, at least in the short run. The results here suggest that even though this 'green paradox' effect of possible innovation can be found both in the non-cooperative case where strategic interaction issues exist and in the cooperative case where strategic interactions do not exist, the increase in initial carbon emissions can be less remarkable in the presence of strategic interactions of carbon taxation and energy pricing between the energy producer side and the consumer side, given that the environmental damage of cumulative emissions is sufficiently high. This result indicates that the 'green paradox' effect of possible innovation can be somewhat restrained by the presence of strategic interactions (rent contest) between resource producers and consumers.

4 Optimal R&D investment

4.1 R&D investment by the consumers

The hazard rate for the innovation of a carbon-free technology to occur at a particular time is exogenously given in the previous sections. In reality, the probability of technology breakthroughs will depend on the R&D efforts of players. Given that the new technology (cheap and clean) will eat the profits of producers, it is reasonable to assume that only the consumer side will make an effort in the R&D of this new technology. Therefore, in this section, the consumers' coalition is allowed to affect the time of innovation by investing in R&D starting from time 0, thereby making the hazard rate of innovation θ a function of the consumer coalition's R&D effort, y. The instantaneous cost of R&D effort is denoted as $C(y)$. To make things simple, let us assume $\theta(y) \equiv y$ and $C(y) \equiv y^2$.[6]

At time 0, the consumers' coalition will choose the optimal R&D effort to maximize its (expected) welfare, taking into account the cost of R&D efforts:

$$\max_{y \geq 0} \; W(0, S_0, y) - \int_0^{+\infty} e^{-rt} [\int_t^{+\infty} h(t_I) dt_I] C(y) dt, \tag{30}$$

where $W(0, S_0, y)$ is the value function for the consumers' coalition (evaluated at ini-

[6]This assumption is not uncommon in the literature; see e.g., Bahel (2011).

tial CO_2 concentration $S(0) = S_0$) obtained in Section 2.2, which is a function of the hazard rate of innovation $\theta$and thus a function of the R&D effort$y$ (since $\theta \equiv y$), and t_I is the instant of time at which the innovation is made. Since t_I is random, one needs to consider the probability that the innovation has not been made by a spacific instant of time (after the innovation, there is no need to undertake R&D any more). $\int_t^{+\infty} h(t_I)d_I = 1 - H(t) = e^{-yt}$ is the probability that the innovation has not been made by time t. $\int_0^{+\infty} e^{-rt}[\int_t^{+\infty} h(t_I)dt_I]C(y)dt$ is thus the total expected effort cost for R&D investment (which has been discounted to time $t = 0$). By integration, one can further find that $\int_0^{+\infty} e^{-rt}[\int_t^{+\infty} h(t_I)dt_I]C(y)dt = \dfrac{y^2}{r+y}$.

The first order condition (interior solution) for the maximization problem (30) is:

$$\frac{\partial W(0, S_0, y)}{\partial y} = \frac{y^2 + 2ry}{(r+y)^2}. \tag{31}$$

The left-hand side of (31) is the marginal benefit of R&D effort and the right-hand side is the marginal cost. It should be noted that the marginal cost of R&D effort at $y = 0$ is equal to zero (i.e., $\left.\frac{y^2+2ry}{(r+y)^2}\right|_{y=0} = 0$). Therefore, if the marginal benefit at $y = 0$ is greater than zero (i.e., $\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0} > 0$), one can conclude that (with the satisfied second-order condition) a positive R&D effort is worth exerting by the consumers' coalition.

Based on the value function for the consumers' coalition (evaluated at the initial concentration level S_0) $W(0, S_0) = w_0 + w_1 S_0 + \frac{1}{2} w_2 [S_0]^2$, where w_0, w_1 and w_2 are functions of θ (thus functions of R&D effort$y$) as in Table 1, we can obtain$\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0} = \frac{\partial w_0(y=0)}{\partial y} + \frac{\partial w_1(y=0)}{\partial y} S_0 + \frac{1}{2}\frac{\partial w_2(y=0)}{\partial y}[S_0]^2$. Since $\frac{\partial z(\theta=0)}{\partial \theta} < 0$ (see Appendix A1) and $\frac{\partial x(\theta=0)}{\partial \theta} > 0$ (see Appendix A2), one can have the following judgement based on expressions for w_2 and w_1 in Table 1: $\frac{\partial w_2(y=0)}{\partial y} = \frac{1}{3}\frac{\partial z(y=0)}{\partial y} < 0$ and $\frac{\partial w_1(y=0)}{\partial y} = \frac{1}{3}\frac{\partial x(y=0)}{\partial y} > 0$ (remember $\theta \equiv y$).

Furthermore, it can be found from the expression for w_0 in Table 1 (making use of $x = \frac{4r\pi^c}{4r+3b(c-z)} - \pi^c$ and $\theta \equiv y$) that:

$$\frac{\partial w_0(y=0)}{\partial y} = \underbrace{-\frac{b}{8r^2}\left[\frac{4r\pi^c}{4r+3b(c-z|_{y=0})}\right]^2}_{<0} \underbrace{+\frac{b}{4r}\left[\frac{4r\pi^c}{4r+3b(c-z|_{y=0})}\right]\frac{\partial x(y=0)}{\partial y}}_{>0} \underbrace{+\frac{\bar{u}}{r^2}}_{>0}$$
$$> -\frac{b}{8r^2}\left[\frac{4r\pi^c}{4r+3b(c-z|_{y=0})}\right]^2 + \frac{\bar{u}}{r^2} > -\frac{b}{8r^2}(\pi^c)^2 + \frac{\bar{u}}{r^2}.$$

Recall that $\bar{u} = \frac{1}{2}a\pi^c + \frac{1}{2}b(p_N)^2 - ap_N$, and we have $\bar{u} \to \frac{1}{2}a\pi^c$ if $p_N \to 0$, which implies that if $p_N \to 0$, then$-\frac{b}{8r^2}(\pi^c)^2 + \frac{\bar{u}}{r^2} \to -\frac{a}{8r^2}\pi^c + \frac{a}{2r^2}\pi^c = \frac{3a}{8r^2}\pi^c$. Consequently, one knows that $\frac{\partial w_0(y=0)}{\partial y} > 0$ if $p_N \to 0$. That is, if the carbon-free technology is sufficiently cheap, we have $\frac{\partial w_0(y=0)}{\partial y} > 0$. Besides, one can easily verify that $\lim_{y\to+\infty}\frac{\partial z}{\partial y} = 0$ (thus $\lim_{y\to+\infty}\frac{\partial w_2}{\partial y} = 0$), $\lim_{y\to+\infty}\frac{\partial x}{\partial y} = 0$ (thus $\lim_{y\to+\infty}\frac{\partial w_1}{\partial y} = 0$), and $\lim_{y\to+\infty}\frac{\partial w_0}{\partial y} = 0$, which implies that $\lim_{y\to+\infty}\frac{\partial W(0,S_0,y)}{\partial y} = 0$. Based on these calculations, the following proposition can be established and demonstrated.

Proposition 5 *With a sufficiently low price for the new carbon-free technology, it would be in the best interest of the consumers' coalition to exert a positive R&D effort on the new technology for any initial CO_2 concentration $0 \leq S_0 < S_\infty$. The optimal R&D effort y^* is an increasing function of the initial CO_2 concentration level: the higher the initial CO_2 concentration level, the greater the optimal R&D effort.*

Proof. As mentioned above, the marginal benefit of R&D effort by the consumers' coalition (evaluated at zero effort) is given by:

$$\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0} = \frac{\partial w_0(y=0)}{\partial y} + \frac{\partial w_1(y=0)}{\partial y}S_0 + \frac{1}{2}\frac{\partial w_2(y=0)}{\partial y}[S_0]^2,$$

where S_0 is the initial CO_2 concentration level.

One can further find that $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0}|_{y=0} = \frac{\partial w_1(y=0)}{\partial y} + \frac{\partial w_2(y=0)}{\partial y} S_0$. It is shown in Appendix A3 that $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0}|_{y=0} > 0$ holds for all $0 \leq S_0 < S_\infty$, which implies that $\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0}$ is an increasing function of the initial CO_2 concentration levelS_0 for $0 \leq S_0 < S_\infty$. It has been shown that, with a sufficiently low price for the new technology, we have $\frac{\partial w_0(y=0)}{\partial y} > 0$. Given that $\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0}$ is an increasing function of S_0 for $0 \leq S_0 < S_\infty$, we have $\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0} \geq \frac{\partial W(0,S_0,y)}{\partial y}|_{y=0,S_0=0} = \frac{\partial w_0(y=0)}{\partial y}$, thereby making $\frac{\partial W(0,S_0,y)}{\partial y}|_{y=0} > 0$ hold for any initial CO_2 concentration $0 \leq S_0 < S_\infty$, which implies that, if the price for the new technology is sufficiently low, the marginal benefit of R&D effort evaluated at zero effort would be always positive.

Recall that the marginal cost of R&D effort at $y = 0$ is zero (i.e., $\left.\frac{y^2+2ry}{(r+y)^2}\right|_{y=0} = 0$). Therefore, with a sufficiently low price for the new technology, the marginal benefit of R&D effort is greater than it marginal cost at $y = 0$. However, if $y \to +\infty$, the marginal benefit of R&D effort is lower than the marginal cost (recall that $\lim_{y \to +\infty} \frac{\partial W(0,S_0,y)}{\partial y} = 0$). Therefore, continuity requires that there exists a $y^* > 0$, such that the first order condition (31) holds and the second order condition for maximization is satisfied. This implies that it is in the best interest of the consumers' coalition to exert a positive R&D effort for any initial CO_2 concentration level $0 \leq S_0 < S_\infty$.

It can also be shown that the optimal R&D effort that the consumers' coalition should exert for inventing the new technology is closely related to the initial CO_2 concentration level. More specifically, recall from (31) that the first order condition for optimal R&D effort $y^*$is $\frac{\partial W(0,S_0,y^*)}{\partial y^*} - \frac{(y^*)^2+2ry^*}{(r+y^*)^2} = 0$, which defines an implicit function $G(S_0, y^*) = 0$. The second-order condition for the maximization of (30) implies that $\frac{\partial^2 W(0,S,y^*)}{\partial^2 y^*} - \frac{2r^2}{(r+y^*)^3} < 0$, i.e., $\frac{\partial G(S_0,y^*)}{\partial y^*} < 0$. Applying the implicit function theorem, we have:

$$\frac{\partial y^*}{\partial S_0} = -\frac{\partial G(S_0, y^*)}{\partial S_0} / \frac{\partial G(S_0, y^*)}{\partial y^*}.$$

It has been demonstrated in Appendix A3 that $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0} > 0$ holds for $0 \leq S_0 < S_\infty$, which implies $\frac{\partial G(S_0,y^*)}{\partial S_0} = \frac{\partial^2 W(0,S_0,y^*)}{\partial y^* \partial S_0} > 0$. Since $\frac{\partial G(S_0,y^*)}{\partial y^*} < 0$ and $\frac{\partial G(S_0,y^*)}{\partial S_0} > 0$, one can know that $\frac{\partial y^*}{\partial S_0} = -\frac{\partial G(S_0,y^*)}{\partial S_0} / \frac{\partial G(S_0,y^*)}{\partial y^*} > 0$, which implies that the optimal R&D effort for the consumers' coalition is an increasing function of the CO_2 concentration level, i.e., the higher the initial CO_2 concentration level, the greater the optimal R&D effort. ■

The result suggests that the urgency to invest in R&D for stimulating innovation in the carbon-free technology would be greater if the starting CO_2 concentration level is found to be higher. This reflects the fact that the innovation of the new technology, which brings no emissions, would prevent the environment from being worsened by further CO_2 emissions from fossil fuels.

4.2 Global efficient R&D investment

The optimal R&D by the consumers' coalition only takes into account its own welfare and ignores the producers' profits, which implies that it is not the global efficient R&D investment. The achievement of global efficient R&D would require a global planner rather than the consumers' collation to make decisions on R&D investment. Specifically, the global planner will solve the following maximization problem to choose the optimal R&D:

$$\max_{y \geq 0} \quad M(0, S_0, y) - \int_0^{+\infty} e^{-rt} [\int_t^{+\infty} h(t_I) dt_I] C(y) dt, \tag{32}$$

where $M(0, S_0, y)$ is the value function (evaluated at initial CO_2 concentration$S(0) = S_0$) for the global planner obtained in Section 2.3, which is a function of hazard rate θ and thus a function of R&D effort y. The first order condition (interior solution) implies:

$$\frac{\partial M(0, S_0, y)}{\partial y} = \frac{y^2 + 2ry}{(r + y)^2}. \tag{33}$$

Similar to the case when the consumers' coalition is making the R&D decision, if the marginal benefit of R&D at $y = 0$ is greater than zero (i.e., $\frac{\partial M(0,S_0,y)}{\partial y}|_{y=0} > 0$), one can conclude that (with the satisfied second-order condition) a positive R&D effort is worth exerting by the global planner. Since $M(0, S_0) = w_0 + w_1 S_0 + \frac{1}{2} w_2 [S_0]^2$, where m_0,$m_1$and m_2 are functions of θ ($\theta \equiv y$) as in Table 2, we can obtain$\frac{\partial M(0,S_0,y)}{\partial y}|_{y=0} = \frac{\partial m_0(y=0)}{\partial y} + \frac{\partial m_1(y=0)}{\partial y} S_0 + \frac{1}{2} \frac{\partial m_2(y=0)}{\partial y} [S_0]^2$. It has been shown in Appendix B1 and B2 that $\frac{\partial m_2(\theta=0)}{\partial \theta} < 0$and $\frac{\partial m_1(\theta=0)}{\partial \theta} > 0$. Moreover, from the expression for m_0 in Table 2 and after some calculations we have:

$$\begin{aligned}
\frac{\partial m_0(y = 0)}{\partial y} &= \underbrace{-\frac{b}{2r^2} \left[\frac{r\pi^c}{r + b(c - m_2|_{y=0})} \right]^2}_{<0} \underbrace{+\frac{b}{r} \left[\frac{r\pi^c}{r + b(c - m_2|_{y=0})} \right] \frac{\partial m_1(y = 0)}{\partial y}}_{>0} \underbrace{+\frac{\bar{u}}{r^2}}_{>0} \\
&> -\frac{b}{2r^2} \left[\frac{r\pi^c}{r + b(c - m_2|_{y=0})} \right]^2 + \frac{\bar{u}}{r^2} > -\frac{b}{2r^2} (\pi^c)^2 + \frac{\bar{u}}{r^2}.
\end{aligned}$$

Following similar reasoning to that of $\frac{\partial w_0(y=0)}{\partial y} > 0$ if $p_N \to 0$, one can easily show $\frac{\partial m_0(y=0)}{\partial y} > 0$ if $p_N \to 0$. That is, if the price of the carbon-free technology is sufficiently low, $\frac{\partial m_0(y=0)}{\partial y} > 0$ will hold.

Besides, one can easily verify that $\lim\limits_{y \to +\infty} \frac{\partial m_2}{\partial y} = 0$, $\lim\limits_{y \to +\infty} \frac{\partial m_1}{\partial y} = 0$, and $\lim\limits_{y \to +\infty} \frac{\partial m_0}{\partial y} = 0$, which implies that $\lim\limits_{y \to +\infty} \frac{\partial M(0,S_0,y)}{\partial y} = 0$. Therefore, following a similar procedure as that in the proof of Proposition 5 and making use of the results in Appendix B3 that $\frac{\partial^2 M(0,S_0,y)}{\partial y \partial S_0}|_{y=0} > 0$ holds for all $0 \le S_0 < S_\infty$, we can show that it is also always optimal for a global planner to invest in R&D, if the price of new technology is sufficiently low, and that the global efficient R&D investment (denoted as y^{**}) is an increasing function of initial CO2 concentration as well.[7]

4.3 Consumers' R&D VS global efficient R&D

We have known that it is always optimal to choose a positive R&D effort for both the consumers' coalition and a global planner, if the price of the new technology is sufficiently low, and that the optimal R&D efforts for both of them should be larger if the initial CO_2 concentration level is higher. However, it might be of great interest to investigate the relative magnitude of optimal R&D investment in the two cases. That is, how large is the optimal R&D investment for the consumers' coalition compared with the global efficient investment? Recall from (31) and (33) that the marginal cost function of the R&D investment is the same for the two cases. Therefore, we simply need to compare the marginal benefit of R&D investment in the case where the consumers' coalition is making the R&D decision with that in the case where the global planner makes the R&D decision instead. By doing this, the following proposition can be established and demonstrated.

Proposition 6 *If the environment damage is sufficiently high, the consumers' coalition will tend to over-invest in R&D for the new technology, compared with the global efficient investment.*

Proof. Recall that the marginal benefit of R&D for the consumers' coalition and the global planner are $\frac{\partial W(0,S_0,y)}{\partial y} = \frac{\partial w_0}{\partial y} + \frac{\partial w_1}{\partial y} S_0 + \frac{1}{2}\frac{\partial w_2}{\partial y}[S_0]^2$ and $\frac{\partial M(0,S_0,y)}{\partial y} = \frac{\partial m_0}{\partial y} + \frac{\partial m_1}{\partial y} S_0 +$

[7] The proof is omitted to save space. The complete demonstration is available from the author upon request.

$\frac{1}{2}\frac{\partial m_2}{\partial y}[S_0]^2$ (where y is the R&D effort and we have hazard rate of innovation $\theta \equiv y$), respectively. The difference between these two is:

$$\frac{\partial W(0,S_0,y)}{\partial y}-\frac{\partial M(0,S_0,y)}{\partial y}=\left(\frac{\partial w_0}{\partial y}-\frac{\partial w_0}{\partial y}\right)+\left(\frac{\partial w_1}{\partial y}-\frac{\partial m_1}{\partial y}\right)S_0+\frac{1}{2}\left(\frac{\partial w_2}{\partial y}-\frac{\partial m_2}{\partial y}\right)[S_0]^2.$$

It is not difficult to show after some calculations[8] that as $\varepsilon \to +\infty$, we have $\frac{\partial w_0}{\partial y} \to \frac{\bar{u}}{r^2}$, $\frac{\partial m_0}{\partial y} \to \frac{\bar{u}}{r^2}$, $\frac{\partial w_1}{\partial y} \to 0$, $\frac{\partial m_1}{\partial y} \to 0$ and $\frac{\partial w_2}{\partial y} - \frac{\partial m_2}{\partial y} \to \sqrt{\varepsilon}\cdot sign\left(\frac{b}{r\sqrt{2b(1+\frac{y}{r})}} - \frac{2}{3}\frac{b}{r\sqrt{6b(1+\frac{y}{r})}}\right) \to +\infty$. This implies that we have $\frac{\partial W(0,S_0,y)}{\partial y} - \frac{\partial M(0,S_0,y)}{\partial y} \to +\infty$ as $\varepsilon \to +\infty$. That is, if environmental damage ε is sufficiently high, we can have $\frac{\partial W(0,S_0,y)}{\partial y} - \frac{\partial M(0,S_0,y)}{\partial y} > 0$, which implies that the marginal benefit of R&D for the consumers' coalition will be larger than that for the global planner. Based on the first order conditions (31) and (33), it can be shown that the optimal R&D investment for the consumers' coalition y^* would be higher than the optimal investment for the global planner y^{**}, implying that the consumers' coalition can over-invest in R&D for the new technology, in the sense that its investment is higher than the global efficient level. ■

This result is mainly due to the fact that the consumers' coalition fails to take into account the effect of innovation on the producers' profits when making its R&D decision while the global planner needs to consider the producers' profits in making its R&D decision. Given that the innovation will eat all the profits of the producers, the global planner might have some hesitation in investing the R&D for the new carbon-free technology.

[8]Complete calculations are available from the authors upon request.

5 Concluding remarks and discussion

This paper investigates the outcomes of the strategic interactions between a resource consumers' coalition and a producers' cartel, taking into account the innovation on a carbon-free technology through a dynamic game. Since the arrival time of innovation is uncertain, both players need to make decisions under the expectation that innovation could happen at any time in the future. With the expectation of possible innovation, the consumer' coalition chooses the optimal carbon taxation to maximize consumers' welfare, taking into account the externality of climate change, while the producers' cartel chooses its optimal (wellhead) energy pricing strategy to maximize its profits. Based on the analytic solutions of the game, the effects of possible innovation in the non-cooperative and cooperative cases are investigated, respectively. Besides, the optimal R&D efforts that should be exerted by the consumers' coalition and by a global planner are characterized. Some important findings or policy implications are summarized as follows.

The anticipation of innovation in cheap non-polluting resources will lower the initial carbon tax, (wellhead) energy price, and consumer price, thereby stimulating a higher initial demand for fossil fuels, and thus higher initial emissions. This is in line with the 'green paradox' argument in the literature (e.g., Sinn, 2008) which states that it is possible that the anticipation of a cheap non-polluting renewable resource will lead fossil fuels owners to increase extraction and may have a detrimental effect on climate change. Our results suggest that in the presence of strategic interactions or rent contest between the resource consumers and producers this 'green paradox' effect still exists. However, compared with the cooperative case where there is no strategic interactions or rent contest, the 'green paradox' effect is found to be less dramatic in the presence of strategic interactions. This implies that the strategic interactions between the resource consumers and producers can somewhat retain this 'green paradox' effect.

Regarding the optimal R&D efforts for innovation, it is found that if the price of the new carbon-free technology is sufficiently low, it is in the best interest of both the consumers' coalition and a global planner to undertake R&D at any initial CO_2 concentration level, which implies it will not be too late to start the R&D. Furthermore, the optimal R&D investment should be an increasing function of the initial CO_2 concentration level. That is, the higher the initial CO_2 concentration level, the larger the optimal R&D effort. However, the R&D investment made by the consumers' coalition could be higher than the global efficient level, if the environmental damage is sufficiently large.

The strategic interactions between the resource consumers' carbon taxation and producers' energy pricing strategy can be very complex when incorporating the possible innovation of carbon-free technologies. To simplify this issue, many assumptions have been made in this paper. For instance, though the model presented here provides some insights into the role of innovation and R&D in the strategic interactions of carbon taxation and energy pricing, the R&D decision by the consumers' coalition is not made simultaneously with the decisions for carbon taxation. It would be interesting to obtain both the Markovian strategy for R&D and the Markovian strategies for carbon taxation and energy pricing at the same time, which of course is difficult to solve analytically and may need the assistance of numerical methods. Besides, to reduce the difficulty of finding the equilibrium, the climate module in the dynamic game here is rather simple; in reality, the climate system can be very complex. It would be of great interest to incorporate a more detailed climate module (e.g., a module that takes into account climate sensitivity) into the current model, which could also be a direction for further research.

Appendix

A1.Proof of $\dfrac{\partial z}{\partial \theta} < 0$

From the expression for $z$in Section 2 one can find its derivative with respect to the hazard rate θ ($\theta \geq 0$) as: $\dfrac{\partial z}{\partial \theta} = \dfrac{2}{3b}\Big(1 - \dfrac{1}{2}\dfrac{2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}}\Big)$.

Let us suppose that $\dfrac{2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}} \leq 2$, which implies that:

$\big[2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}\big]^2 \leq 4[(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})]$.

Calculating the square on the left-hand side and reordering terms, we have the contradiction: $9b^2c^2 + 36b^2(\frac{\varepsilon}{r})^2 + 36b^2c\varepsilon\frac{1}{r} \leq 0$, which allows us to establish that:

$\dfrac{2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}} > 2$, i.e., $1 - \dfrac{1}{2}\dfrac{2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}} < 0$, thereby making $\dfrac{\partial z}{\partial \theta} < 0$. One can verify that, as a special case, $\dfrac{\partial z(\theta = 0)}{\partial \theta} < 0$ (also denoted as $\dfrac{\partial z}{\partial \theta}|_{\theta=0} < 0$) will also hold.

A2.Proof of $\dfrac{\partial x}{\partial \theta} > 0$

Taking the derivative of x with respect to the hazard rate of innovation θ ($\theta \geq 0$), after some calculations, one can obtain (complete calculations are available upon request):

$\dfrac{\partial x}{\partial \theta} = \dfrac{8\pi^c}{[4(r+\theta) + 3b(c-z)]^2}\Big(\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})} - \dfrac{r+\theta}{2}\dfrac{2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}}\Big)$.

Let us suppose $\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})} \leq \dfrac{r+\theta}{2}\dfrac{2(r+\theta) + 3bc + 6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r})}}$, which implies that $(r+\theta)^2 + 3bc(r+\theta) + 6b\varepsilon(1+\frac{\theta}{r}) \leq (r+\theta)^2 + \frac{3}{2}bc(r+\theta) + 3b\varepsilon(1+\frac{\theta}{r})$.

Reordering terms we have the contradiction: $\frac{3}{2}bc(r+\theta)+3b\varepsilon(1+\frac{\theta}{r})\leq 0$, which allows us to establish that $\sqrt{(r+\theta)^2+3bc(r+\theta)+6b\varepsilon(1+\frac{\theta}{r})} > \frac{r+\theta}{2}\frac{2(r+\theta)+3bc+6b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+3bc(r+\theta)+6b\varepsilon(1+\frac{\theta}{r})}}$, and therefore we have: $\frac{\partial x}{\partial \theta} > 0$. As a special case, $\frac{\partial x(\theta=0)}{\partial \theta} > 0$ will hold as well.

A3. Proof of $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0} > 0$ for $0 \leq S_0 < S_\infty$

Since $\frac{\partial W(0,S_0,y)}{\partial y} = \frac{\partial w_0}{\partial y} + \frac{\partial w_1}{\partial y}S_0 + \frac{1}{2}\frac{\partial w_2}{\partial y}[S_0]^2$, we have $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0} = \frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial y}S_0 = \frac{\partial(w_1+w_2S_0)}{\partial y}$. Noting that $w_1+w_2S_0 = (w_1+w_2S_\infty)+w_2(S_0-S_\infty)$ and being aware of the relationship $S_\infty = \frac{x+\pi^c}{c-z}$, $w_1 = \frac{1}{3}x$ and $w_2 = z - \frac{4\varepsilon}{r}$, one can obtain the following relationship after some calculations: $w_1+w_2S_0 = -\frac{2\pi^c\varepsilon}{rc+2\varepsilon} + w_2(S_0-S_\infty)$.

Therefore, we have $\frac{\partial(w_1+w_2S_0)}{\partial y} = \frac{\partial w_2}{\partial y}(S_0-S_\infty)$. Since $\frac{\partial w_2}{\partial y} = \frac{1}{3}\frac{\partial z}{\partial y} < 0$ (recall $\theta \equiv y$ and see Appendix A1 for the proof of $\frac{\partial z}{\partial \theta} < 0$), we have $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0} = \frac{\partial(w_1+w_2S_0)}{\partial y} > 0$ for $S_0 < S_\infty$. And it is given that the initial CO_2 concentration $S_0 \geq 0$. Therefore, we have $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0} > 0$ for all $0 \leq S_0 < S_\infty$. As a special case, we have $\frac{\partial^2 W(0,S_0,y)}{\partial y \partial S_0}|_{y=0} > 0$ for $0 \leq S_0 < S_\infty$.

B1. Proof of $\frac{\partial m_2}{\partial \theta} < 0$

It is not difficult to find the derivative: $\frac{\partial m_2}{\partial \theta} = \frac{1}{2b}\Big(1 - \frac{1}{2}\frac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}}\Big)$.

Let us suppose that $\frac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}} \leq 2$, which implies that:

$\big(2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}\big)^2 \leq 4[(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})]$.

Calculating the square on the left-high side and reordering terms we have the contradiction: $16b^2c^2+64b^2(\frac{\varepsilon}{r})^2+64b^2c\varepsilon\frac{1}{r} \leq 0$, which allows us to establish that:

$\frac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}} \geq 2$, i.e., $1-\frac{1}{2}\frac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}} < 0$, thereby making $\frac{\partial m_2}{\partial \theta} < 0$

B2. Proof of $\dfrac{\partial m_1}{\partial \theta} > 0$

Taking the derivative of m_1 with respect to the hazard rate of innovation θ, and after some calculations (which are available the author upon request), one can obtain: $\dfrac{\partial m_1}{\partial \theta} = \dfrac{\pi^c}{2[(r+\theta)+b(c-m_2)]^2}\Big(\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})} - \dfrac{r+\theta}{2}\dfrac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}}\Big)$.

Let us suppose $\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})} \leq \dfrac{r+\theta}{2}\dfrac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}}$, which implies that $(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r}) \leq (r+\theta)^2+2(r+\theta)bc+4b\varepsilon(1+\frac{\theta}{r})$. Reordering terms we have the contradiction: $2bc(r+\theta)+4b\varepsilon(1+\frac{\theta}{r}) \leq 0$, which allows us to establish $\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})} > \dfrac{r+\theta}{2}\dfrac{2(r+\theta)+4bc+8b\varepsilon\frac{1}{r}}{\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}}$, and therefore we have: $\dfrac{\partial m_1}{\partial \theta} > 0$.

B3. Proof of $\dfrac{\partial^2 M(0,S_0,y)}{\partial y \partial S_0} > 0$ for $0 \leq S_0 < S_\infty$

Since $\dfrac{\partial M(0,S_0,y)}{\partial y} = \dfrac{\partial m_0}{\partial y} + \dfrac{\partial m_1}{\partial y}S_0 + \dfrac{1}{2}\dfrac{\partial m_2}{\partial y}[S_0]^2$, we have $\dfrac{\partial^2 M(0,S_0,y)}{\partial y \partial S_0} = \dfrac{\partial m_1}{\partial y} + \dfrac{\partial m_2}{\partial y}S_0 = \dfrac{\partial (m_1+m_2S_0)}{\partial y}$. Noting that $m_1+m_2S_0 = (m_1+m_2S_\infty)+m_2(S_0-S_\infty)$ and being aware of the relationship $S_\infty = S_\infty^G = \dfrac{\pi^c+m_1}{c-m_2}$, $m_1 = \dfrac{(r+\theta)\pi^c}{(r+\theta)+b(c-m_2)} - \pi^c$ and $m_2 = c + \dfrac{1}{2b}\Big(r+\theta - \sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\frac{\theta}{r})}\Big)$, after some calculations, one can obtain: $m_1+m_2S_0 = -\dfrac{2\pi^c\varepsilon}{rc+2\varepsilon} + m_2(S_0-S_\infty)$.

Therefore, we have $\dfrac{\partial (m_1+m_2S_0)}{\partial y} = \dfrac{\partial m_2}{\partial y}(S_0-S_\infty)$. Recalling $\theta \equiv y$ and $\dfrac{\partial m_2}{\partial \theta} < 0$ (see Appendix B1), we have $\dfrac{\partial^2 M(0,S_0,y)}{\partial y \partial S_0} = \dfrac{\partial (m_1+m_2S_0)}{\partial y} > 0$ for $S_0 < S_\infty$. It is given that the initial CO_2 concentration $S_0 \geq 0$. Therefore, we have $\dfrac{\partial^2 M(0,S_0,y)}{\partial y \partial S_0} > 0$ for all $0 \leq S_0 < S_\infty$. As a special case, we have $\dfrac{\partial^2 M(0,S_0,y)}{\partial y \partial S_0}|_{y=0} > 0$ for $0 \leq S_0 < S_\infty$.

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Table 1. Coefficients for value functions $W(0,S)$ and $V(0,S)$

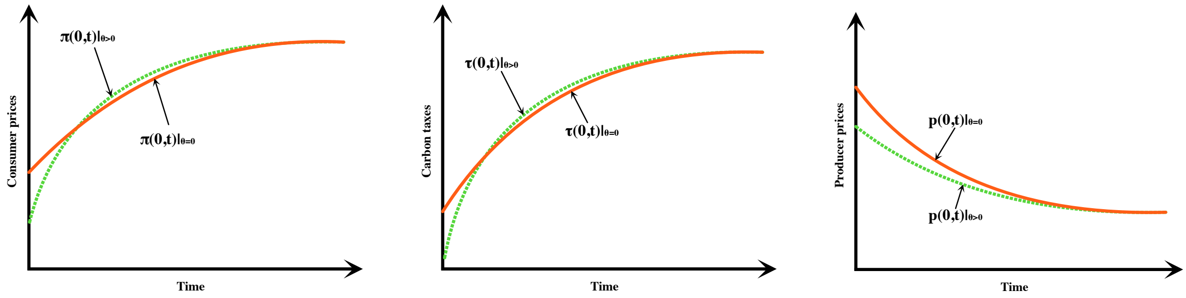
$w_0 = \frac{b}{8(r+\theta)}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)}\right]^2 + \frac{\theta\bar{u}}{r(r+\theta)}$	$v_0 = \frac{b}{4(r+\theta)}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)}\right]^2$
$w_1 = \frac{1}{3}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)} - \pi^c\right] = \frac{1}{3}x$	$v_1 = \frac{2}{3}\left[\frac{4(r+\theta)\pi^c}{4(r+\theta)+3b(c-z)} - \pi^c\right] = \frac{2}{3}x$
$w_2 = \frac{1}{3}\left[z - \frac{4\varepsilon}{r}\right]$	$v_2 = \frac{2}{3}\left[z + \frac{2\varepsilon}{r}\right]$

Table 2. Coefficients for value function $M(0,S)$

$$m_0 = \frac{b}{2(r+\theta)}\left[\frac{(r+\theta)\pi^c}{(r+\theta)+b(c-m_2)}\right]^2 + \frac{\theta\bar{u}}{r(r+\theta)}$$

$$m_1 = \frac{(r+\theta)\pi^c}{(r+\theta)+b(c-m_2)} - \pi^c$$

$$m_2 = c + \frac{1}{2b}\left(r+\theta-\sqrt{(r+\theta)^2+4bc(r+\theta)+8b\varepsilon(1+\tfrac{\theta}{r})}\right)$$



a. Consumer price b. Carbon tax c. Producer price

Figure 1. Temporary trajectories for players' strategies with/without the innovation possibility