Discounting and Relative Consumption

OLOF JOHANSSON-STENMAN AND THOMAS STERNER

Motivation

The theory and practice of discounting is essential for dealing with long-term phenomena

With the increased attention to climate change, interest in discounting has experienced a revival

(e.g. Gollier 2010, Gollier and Weitzman 2010, and Weitzman 2010)

Virtually the entire economics debate in the wake of the *Stern Review* focused on the discount rate used

(Brekke and Johansson-Stenman 2008; Dasgupta 2007, 2008; Dietz and Stern 2007; Heal 2008; Howarth 2009; Nordhaus 2007; Sterner and Persson 2008; Weitzman 2007a, 2007b)

The reason is obvious:

Much of the consequences of climate change will occur far into the future and, thus, the discount rate will have a dramatic effect on their present value

That humans value consumption in relation to others' consumption was noted already by Adam Smith, John Stuart Mill and Alfred Marshall

There is also much recent empirical evidence

This paper, as far as we know, is the first to incorporate relative consumption effects in the theory of social discounting

To determine how such concerns affect social discounting is the aim of the paper

The conventional Ramsey discounting rule

The standard model of discounting is typically derived in general equilibrium, with the following objective function:

$$\max_{c_\tau} w \equiv \int_0^T v\left(c_\tau\right) e^{-\delta\tau} d\tau$$

The discount rate is then given by

$$\rho_t = -\frac{\partial\left(\partial w / \partial c_t\right) / \partial t}{\partial w / \partial c_t}$$

which can be written as

$$\rho_t = \delta + \sigma_t g_t$$

where $\sigma_t \equiv -c_t v_{11t} / v_{1t}$ is the individual coefficient of relative risk aversion

Discounting when people care about relative consumption

Following Arrow and Dasgupta (2009), society consists of identical individuals with a population size normalized to one

In addition to absolute consumption, they also care about relative consumption

$$R_t = r(c_t, z_t)$$

where z_t is the average of others' consumption

We assume that *R* is unaffected if own consumption and others' consumption are changed equally, i.e. $r_{1t} = -r_{2t}$

This encompasses the most commonly used comparison forms, the difference comparison form where

$$R_t = c_t - z_t$$

and the ratio comparison case where

$$R_t = c_t / z_t$$

The individual instantaneous utility (or felicity) at time *t* is:

$$U_t = u\left(c_t, R_t\right) = u\left(c_t, r(c_t, z_t)\right) = v\left(c_t, z_t\right)$$

In addition, we make the common assumption of (weak) keeping-up-with-the-Joneses property, such that $v_{12t} \geq 0$.

Will the optimal discounting rule change when we introduce relative consumption concerns?

The answer is not obvious

Note that people care about relative consumption now, but also in the future

Note also that we have now two different discount rates, the private and the social one

The individual's maximization problem:

$$\max_{c_\tau} w^p \equiv \int_0^T u\left(c_\tau, r(c_\tau, z_\tau)\right) e^{-\delta\tau} d\tau = \int_0^T v\left(c_\tau, z_\tau\right) e^{-\delta\tau} d\tau \quad .$$

Hence, the individual takes others' consumption, z_t, as given

The social maximization problem:

$$\max_{c_\tau} w^s \equiv \int_0^T u\left(c_\tau, r(c_\tau, c_\tau)\right) e^{-\delta\tau} d\tau = \int_0^T v\left(c_\tau, c_\tau\right) e^{-\delta\tau} d\tau$$

Hence, relative consumption, R_t, is taken to be given

Let us introduce a measure of how much relative consumption matter:

The *degree of positionality* is defined by:

$$\gamma_t \equiv \frac{u_{2t}\, r_{1t}}{u_{1t} + u_{2t} r_{1t}}$$

Proposition For a positive growth rate $\rho^{s}(t) > (<) \rho^{p}(t) \ \forall t$ if and only if $\partial\gamma_t / \partial c_t > (<) \ 0 \ \forall t$

Hence, if relative consumption becomes more important (compared to absolute consumption) when consumption increases, then the social discount rate is larger than the private one

Two different measures of relative risk aversion

The *individual* coefficient of relative risk aversion:

$$\sigma_t \equiv -c_t v_{11t} / v_{1t}$$

Others' consumption is held fixed

The *social* coefficient of relative risk aversion:

$$\psi_t \equiv -c_t u_{11t} / u_{1t}$$

Relative consumption is held fixed; can be thought of as related to a lottery for all individuals simultaneously

Comparison with the Ramsey rule

Proposition. For a positive growth rate, $\rho_t^s > (<) \rho_t^R$ if and only if $\psi_t > (<) \sigma_t$

Hence, the social discount rate exceeds the Ramsey discount rate if the social coefficient of relative risk aversion exceeds the individual coefficient of relative risk aversion

But when is this the case? We have some ideas about the size of σ_t, but not really about ψ_t

Yet another measure of relative risk aversion

The coefficient of reference consumption relative risk aversion is given by:

$$\theta_t \equiv z_t v_{22t} / v_{2t}$$

and reflects risk aversion with respect to others' consumption

Do I prefer that others have 100 for sure or a 50/50 lottery where they either get 50 or 150?

The elasticity of substitution between c_t and z_t is given by:

$$\Phi_t \equiv \frac{-\frac{v_{11t}}{v_{1t}^{\;2}} - \frac{v_{22t}}{v_{2t}^{\;2}} + 2\frac{v_{12t}}{v_{1t}v_{2t}}}{\frac{1}{v_{1t}c_t} + \frac{1}{v_{2t}z_t}} = \left(-\frac{v_{11t}}{v_{1t}^{\;2}} - \frac{v_{22t}}{v_{2t}^{\;2}} + 2\frac{v_{12t}}{v_{1t}v_{2t}} \right) \frac{v_{1t}v_{2t}}{v_{1t} + v_{2t}} c_t \, .$$

Proposition. The social discount rate can be written as:

$$\rho_t^s = \delta + \left(\sigma_t - \theta_t - \Phi_t\right) g_t = \rho_t^R - \left(\theta_t + \Phi_t\right) g_t$$

Hence, $\rho_t^s < \rho_t^R$ iff $\theta_t + \Phi_t > 0$

Thus, given quasiconcavity and reference consumption risk aversion, the social discount rate is smaller than the Ramsey discount rate

Corollary. For a positive growth rate, if $d\gamma_t / dc_t > 0$ and $\theta_t + \Phi_t > 0$ then $\rho_t^p < \rho_t^s < \rho_t^R$

Order of magnitudes

Consider a functional form similar to Dupor and Liu (2003):

$$U_t = \frac{1}{1-\alpha}\left((1-a)c_t^{1-\omega} + a\left(c_t^{1-\omega} - z_t^{1-\omega}\right)\right)^{\frac{1-\alpha}{1-\omega}} = \frac{1}{1-\alpha}\left(c_t^{1-\omega} - a z_t^{1-\omega}\right)^{\frac{1-\alpha}{1-\omega}}$$

- The degree of positionality is $\gamma_t = a$
- The elasticity of substitution between own consumption and reference consumption is ω

- The individual coefficient of relative risk aversion is

$$\sigma_t = \frac{\alpha - \omega a}{1 - a}$$

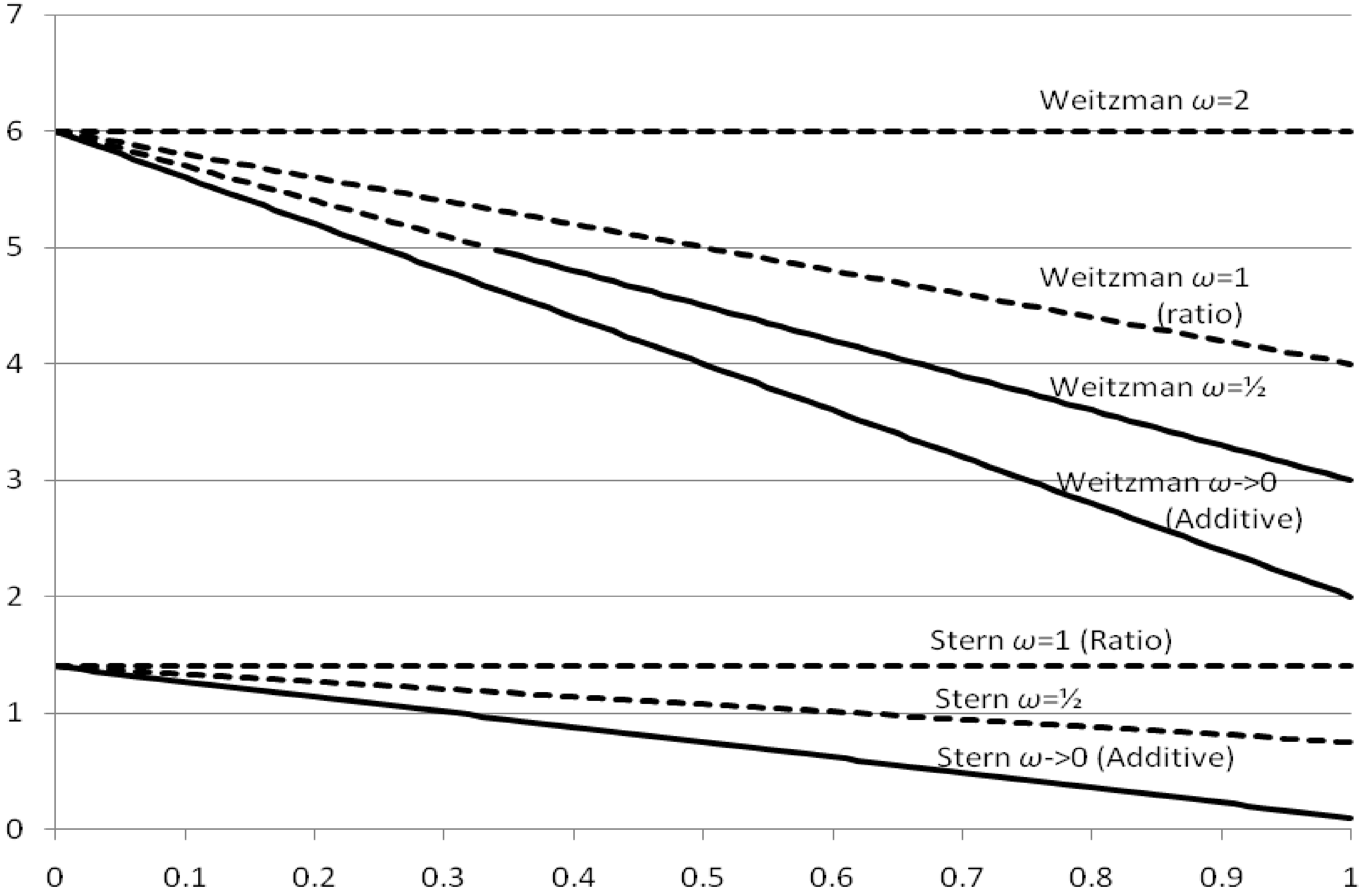
- The social coefficient of relative risk aversion is $\psi_t = \alpha$
- The coefficient of reference-consumption relative risk aversion is $\theta_t = \dfrac{a\alpha - \omega}{1 - a}$

When $\omega = 0$, we obtain the simple *difference comparison form* so that own consumption and (the negative of) others' consumption are perfect substitutes:

$$U_t = \frac{1}{1-\alpha}\Big((1-a)c_t + a\big(c_t - z_t\big)\Big)^{(1-\alpha)} = \frac{1}{1-\alpha}\big(c_t - az_t\big)^{(1-\alpha)}$$

Similarly, we obtain the *ratio-comparison form* by letting ω approach unity:

$$U_t = \frac{1}{1-\alpha}\left(c_t\left(\frac{c_t}{z_t}\right)^{a/(1-a)}\right)^{(1-\alpha)} = \frac{1}{1-\alpha}c_t^{(1-\alpha)/(1-a)}z_t^{-(1-\alpha)a/(1-a)}$$



7
6
5
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Weitzman ω=2
Weitzman ω=1 (ratio)
Weitzman ω=½
Weitzman ω->0 (Additive)
Stern ω=1 (Ratio)
Stern ω=½
Stern ω->0 (Additive)

Conclusion and Discussion

Several reasons in the literature suggest that the social discount rate should be lower than the private discount rate:

For example, individuals are more risk averse than society in the presence of uncertainty, and societal time horizons are longer than individual ones (cf. Arrow and Lind 1970)

In this paper, we show that relative consumption effects do *not* provide another reason

On the contrary, the social discount rate tends, under positional concern, to exceed the private one, provided that the degree of positionality increases as we get richer

Yet, from a climate policy perspective, it is more important whether the optimal social discount rate should be modified, compared to the conventional Ramsey rule

We show that, for a positive growth rate, the social discount rate is smaller than the Ramsey discount rate if preferences are quasi-concave in own and reference consumption and exhibit risk aversion with respect to reference consumption

We also demonstrate numerically that the discrepancies may be substantial, although the underlying parameter estimates are uncertain

The impact of the discount interest rates on the economics of long-term phenomena, such as global warming, is large even for modest adjustments of the discount rate

Thus, overall it is fair to conclude that taking relative consumption effects into account may have a profound effect

Thank you for listening!